An attention-based explanation for some exhaustivity operators*

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Abstract. A well-known challenge for accounts of exhaustivity implications is the *granularity problem*: that adding a non-weakest disjunct to an utterance (e.g., "or both") may prevent exhaustivity implications. Recent approaches to this problem apply exhaustivity operators either globally, i.e., to the disjunction as a whole, or locally, i.e., to each disjunct separately. This paper seeks to contribute to a better understanding of the operators employed in the globalist strand, which, contrary to globalists' aims, have not thus far been given any sort of pragmatic motivation. To that end this paper demonstrates that these operators can be derived, wholly or in part, from a pragmatic theory: *Attentional Pragmatics* (Westera, 2017). The theory centers on the assumption that speakers should not only assert all relevant propositions they hold true, but also draw attention to all relevant propositions they consider possible. This assumption, suitably formalized, overcomes the granularity problem. The current paper formally derives an exhaustivity operator from Attentional Pragmatics and proves that it is in important respects conservative with regard to existing operators.

Keywords: exhaustivity, Hurford disjunction, granularity, Attentional Pragmatics, globalism/localism, innocent exclusion, minimal worlds.

1. Introduction

Adding a non-weakest disjunct can affect the exhaustivity implications of an utterance:

(1) A: Who (of John, Mary and Bill) is at the party?
  a. B: John is there. *(implied: not Mary, not Bill)*
  b. B: John is there, or both John and Mary. *(implied: ~~not Mary,~~ not Bill)*

This poses a challenge for traditional pragmatic accounts of exhaustivity, which are based on considerations of informational strength, i.e., the Gricean maxim of Quantity (e.g., Horn 1972; Gazdar 1979; Schulz and Van Rooij 2006; Spector 2007; Geurts 2011). The reason is that considerations of informational strength alone are arguably unable to distinguish between examples like (1a) and (1b). After all, the most straightforward semantic informational contents (i.e., literal sentence meanings) and hence primary informational intents (i.e., speaker meanings) for (1a) and (1b) would correspond to the classical meanings of their closest translations into predicate logic, Pj and $Pj \vee (Pj \wedge Pm)$ – but these are classically (informationally) equivalent. I will call this the **granularity problem**: the traditional, information-based pragmatic approach is too coarse-grained to see the difference between (1a) and (1b).

The granularity problem was noted already by Gazdar (1979), and several approaches have been explored. Gazdar tried to solve it by assuming that utterances have "clausal" implications, to the effect that a speaker should be uncertain about any embedded clause of an uttered

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sentence, e.g., the disjuncts of a disjunction. However, besides overgenerating clausal implications, the pragmatic explanation for clausal implications relies on the assumption that both embedded clauses and their negations are always relevant, a type of symmetry that is not plausible (Horn, 1989) and that leads to the well-known symmetry problem (Kroch, 1972). More recent approaches can be divided into two main branches:

- **Localist:** exhaustivity operators are applied to each disjunct (e.g., Chierchia et al. 2012), potentially motivated pragmatically by considerations of redundancy (e.g., Katzir and Singh 2013; Ciardelli and Roelofsen 2016).
- **Globalist:** an exhaustivity operator is applied only globally, to the disjunction as a whole, but the operator is a more sophisticated one that somehow has access to the individual disjuncts (Schulz and Van Rooij 2006; Alonso-Ovalle 2008). No pragmatic motivation for these operators has been given.

The current paper aims to mend the gap in the globalist approaches, by outlining a pragmatic motivation for an adequate exhaustivity operator. To that end I adopt the theory of Attentional Pragmatics (Westera 2017), which defines a set of maxims that govern not only information sharing, but also attention sharing – a promising approach because while (1a) and (1b) are informationally equivalent, they are arguably attentionally distinct.

The theory of Attentional Pragmatics is summarized in section 2; section 3 derives a new exhaustivity operator from this theory; and section 4 demonstrates its (partial) conservativeness with regard to existing operators. Section 5 concludes.

2. Attentional Pragmatics (summary of Westera 2017)

2.1. Framework and formalism

For a clear exposition it is necessary that I summarize the conceptual and formal pragmatic framework adopted in Westera 2017. A core assumption is the following:

> **Assumption 1.** Speakers have certain communicative intentions, e.g., to share a piece of information, the objects of which are called "intents".

Intents are to be distinguished from the (semantic) *contents* of a sentence. This distinction is adopted from Bach and Harnish 1979, and it generalizes Grice's (1989) distinction between *speaker meaning* and *sentence meaning*. In this paper I will say only very little about semantic contents; I will just presuppose that an adequate semantics exists which, through the maxim of Manner, delivers the assumed intents. Another core assumption is the following:

> **Assumption 2.** A goal of making a certain piece of information common ground is not pursued on its own, but as part of what is called a "theme": a set of propositions that share a certain subject matter and that each ought to be made common ground.

Themes are more commonly called "questions under discussion", but this gives rise to a potentially harmful confusion between questions as discourse goals, as utterances, as meanings of interrogative sentences and as meanings of embedded interrogative-like constructions.

The assumed intents and themes for the relevant examples, as well as the conversational maxims, will be specified in *Intensional Logic* (Montague, 1973), albeit with doxastic rather than alethic modality and with some additional notation conventions. I refer to the exposition in Gamut 1991 (vol.2) for the basic formalism. As a brief reminder: the operators $^\wedge$ and $^\vee$ signify abstraction over and application to worlds, which will be used in this paper almost exclusively to switch between propositions and their truth values, i.e., $^\wedge\varphi$ can generally be read as "(the proposition) that φ", and $^\vee p$ as "the proposition p is true here". To illustrate, the formula $\Box(Pj \wedge \mathcal{T}_0(^\wedge Pj))$ might express that the speaker takes herself to know that John is at the party and that the proposition that John is at the party is an element of the main theme.

As is common, expressions of certain relevant types will be distinguished typographically. Besides using lowercase for individual constants/variables ($a, b, c, \ldots$ of type e) and uppercase for predicates ($A, B, C, \ldots$ of type $\langle e, t\rangle$), I will use lowercase calligraphic constants/variables for propositions ($a, b, c, \ldots$ of type $\langle s, t\rangle$) and uppercase calligraphic for sets of propositions ($\mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots$ of type $\langle\langle s, t\rangle, t\rangle$). Furthermore, as a notational convention, for all unary, first-order predicate constants P, I may write, e.g., $Pjmb$ to mean $Pj \wedge Pm \wedge Pb$. In addition, it will occasionally be convenient to conceive of functions of type $\langle a, t\rangle$ as sets of things of type a, and to use the usual set-theoretical operations and relations within the object language (these can be defined as mere notational shorthands; Zimmermann 1989). Lastly, for any set-type expression X I will write $X^\cap$ to mean the closure of X under intersection.

In order to formalize the theory, type-theoretical expressions will be interpreted on a subclass of *admissible models*, designed so as to interpret certain constants in a certain way, e.g., the constant I-RELATION will be interpreted basically as the Gricean maxim of (I(nformational)-)Relation (cf. Montague's meaning postulates). Admissible models must also validate the KD45 belief axioms, plus *intent introspection* and *theme introspection*, which ensure that the speaker knows the interpretations of constants denoting intents and themes of the utterance (e.g., p_0 for the main informational intent, i.e., what is asserted) – I leave their definitions implicit. Thus:

Definition 1 (Admissible model)**.** A model $\mathbf{M}$ (or $\langle \mathbf{M}, w_0\rangle$) is an *admissible model* iff:

1. $\mathbf{M}$ validates (makes true in all its worlds) the KD45 belief axioms;
2. $\mathbf{M}$ validates intent and theme introspection; and
3. $\mathbf{M}$ validates the definitions of the maxims, to be given below.

And it is an admissible model *for a given example* if, and only if, in addition:

4. w_0 validates all formal statements given in the example (in a gray box); and
5. W is sufficiently large, namely, every contingent first-order formula, that can be constructed from only constants used in the example, variables, connectives and quantifiers, is true in some $w \in W$.

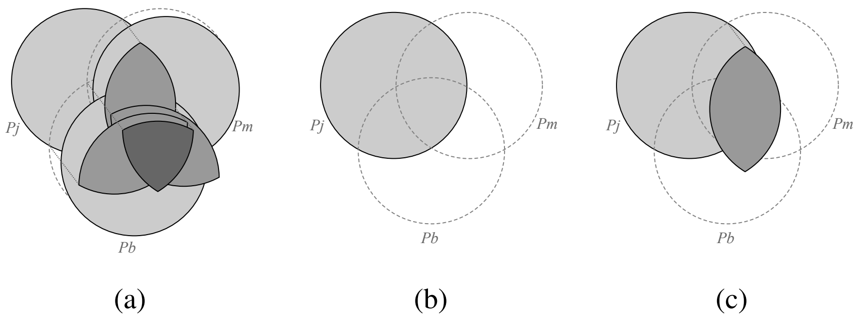


Figure 1: The theme and attentional intents of (2a), (2b) and (2c).

Admissible models for a given example enable us to instantly formalize the relevant parts of a given example and prove potentially interesting things about it, e.g., that an utterance complies with the Gricean maxim of Relation, which is the case if in all admissible models for the example the constant I-RELATION returns true in the actual world, when applied to the relevant intents and themes, and given the beliefs and goals of the speaker.

On top of the common assumption that utterances have themes and intents, we need a particular type of intent to solve the granularity problem (building on Ciardelli et al. 2009):

Assumption 3. Utterances have *attentional intents*. An attentional intent is a non-empty set of propositions to which a speaker intends to draw the audience's attention.

Note that an utterance may draw attention to many things, propositions and otherwise, including (but not limited to) everything explicitly mentioned in the utterance – but not everything will be part of what the speaker *intended* to draw attention to, i.e., part of the attentional intent. Note furthermore that the assumption that utterances have attentional *intents* does not commit one to the assumption of a corresponding dimension of attentional semantic *content*. In Westera 2017 it is shown that attentional intents can typically be clearly conveyed without such an enriched semantic dimension, simply on the basis of the informational contents of all uttered constituents.

To illustrate, and also to get acquainted with the formalism, let us consider the intents and themes of (1) with which this chapter started, repeated here as (2):

(2) a. A: Who (of John, Mary and Bill) is at the party?

$\mathcal{T}_0 = \{{}^\wedge Pj, {}^\wedge Pm, {}^\wedge Pb\}^\cap \qquad \mathcal{A}_0 = \{{}^\wedge Pj, {}^\wedge Pm, {}^\wedge Pb\}^\cap$

b. B: John is at the party.

$\mathcal{T}_0 = \{{}^\wedge Pj, {}^\wedge Pm, {}^\wedge Pb\}^\cap \qquad p_0 = {}^\wedge Pj \qquad \mathcal{A}_0 = \{{}^\wedge Pj\}$

c. B: John is at the party, or John and Mary.

$\mathcal{T}_0 = \{{}^\wedge Pj, {}^\wedge Pm, {}^\wedge Pb\}^\cap \qquad p_0 = {}^\wedge Pj \qquad \mathcal{A}_0 = \{{}^\wedge Pj, {}^\wedge Pjm\}$

For each utterance $\mathcal{T}_0$ denotes the main theme, p_0 the main informational intent (what is asserted) and $\mathcal{A}_0$ the main attentional intent. Recall that $\{...\}^{\cap}$ is a notational shorthand for closure under intersection. For independent motivation of the assumed themes and intents I refer to Westera 2017. The assumed attentional intents of (2a), (2b) and (2c) are depicted, from left to right, in figure 1. This type of pictorial representation will be relied upon again further below. In it, each attentional intent is depicted as a Venn diagram on the set of all possible worlds, based on the three atomic propositions of John's, Mary's, and Bill's presence (the circles). Overlapping propositions in the attentional intent – the gray regions – are pulled apart in a third dimension, towards the reader as it were, for clearer presentation.

2.2. Maxims governing informational intents

The maxims are defined, recall, by fixing the interpretation of a number of designated constants in the class of admissible models. The *I(nformation)-maxims* closely resemble Grice's (1989, ch.2) maxims (except Manner, which will not play an explicit role in this paper). Grice's informal approach can of course be formalized in different ways, and for a motivation of the following definition I refer to Westera 2017 (e.g., it predicts the right distribution of intonational cues of maxim violations; cf. Westera 2013):

Definition 2.

1. $\text{I-Q}{\small\text{UALITY}}(p) = \Box^{\vee} p$
   "Intend to share only information you take to be true."

2. $\text{I-R}{\small\text{ELATION}}(p, \mathcal{T}) = \mathcal{T}(p)$
   "Intend to share only information that is thematic."

3. $\text{I-Q}{\small\text{UANTITY}}(p, \mathcal{T}) = \forall q \left(\left(\begin{array}{c} \text{I-Q}{\small\text{UALITY}}(q) \wedge \\ \text{I-R}{\small\text{ELATION}}(q, \mathcal{T}) \end{array} \right) \rightarrow (p \subseteq q) \right)$
   "Intend to share all thematic information you take to be true."

From this definition we can derive some general results, e.g., that a speaker will always know whether a given intent complies with the I-maxims or not, and that if there exists a compliant intent then it is the only one. We can also compute more concrete predictions, e.g.:

Fact 1. For all admissible models $\langle \mathbf{M}, w_0 \rangle$ for example (2b):

$$\mathbf{M}, w_0 \models \text{I-Q}{\small\text{UANTITY}}(p_0, \mathcal{T}_0) \rightarrow (\neg \Box Pm \wedge \neg \Box Pb)$$

This I-Quantity implication is the starting point of traditional pragmatic accounts of exhaustivity implications. However, since it is (correctly) predicted for (2b) and (2c) alike, it does not provide us with a handle for distinguishing them – this is the granularity problem. Note, furthermore, that the absence of belief ($\neg \Box Pm$) does not entail a belief to the contrary ($\Box \neg Pm$), i.e., exhaustivity. This is the well-known *epistemic step*, which elsewhere I have argued is a genuine empirical problem for the standard, I-Quantity-based recipe for exhaustivity (Westera 2014).

2.3. Maxims governing attentional intents

The *A(ttention)-maxims* follow the same general recipe as the I-maxims, except for the addition of the maxim of A-Parsimony, which I will briefly motivate shortly:

Definition 3.

1. $\text{A-QUALITY}(\mathcal{A}) = \forall a(\mathcal{A}(a) \rightarrow \Diamond^{\vee} a)$
   "Intend to draw attention only to propositions that you consider possible."

2. $\text{A-RELATION}(\mathcal{A}, \mathcal{T}) = \forall a(\mathcal{A}(a) \rightarrow \mathcal{T}(a))$
   "Intend to draw attention only to thematic propositions."

3. $\text{A-PARSIMONY}(\mathcal{A}, \mathcal{T}) = \forall a \left(\begin{array}{c} (\mathcal{A}(a) \wedge \text{A-QUALITY}(\{a\})) \rightarrow \\ \Diamond \left({}^{\vee}a \wedge \forall b \left(\left(\begin{array}{c} b \subset a \wedge \\ \text{A-RELATION}(\{b\}, \mathcal{T}) \end{array} \right) \rightarrow \neg^{\vee} b \right) \right) \end{array} \right)$
   "Intend to draw attention to a proposition only if, if you consider it possible, you consider it possible independently of any more specific thematic proposition(s)."

4. $\text{A-QUANTITY}(\mathcal{A}, \mathcal{T}) = \forall a \left(\left(\begin{array}{c} \text{A-QUALITY}(\{a\}) \wedge \\ \text{A-RELATION}(\{a\}, \mathcal{T}) \wedge \\ \text{A-PARSIMONY}(\{a\}, \mathcal{T}) \end{array} \right) \rightarrow \mathcal{A}(a) \right)$
   "Intend to draw att. to all thematic propositions you consider independently possible."

Again some general results can be proven, e.g., that a speaker will always know whether the A-maxims are complied with, and that if there is a compliant attentional intent then it is the only one. As on the informational side, for a motivation of this particular definition of the maxims I refer to Westera 2017. I will here summarize only the motivation for A-Parsimony. Consider a speaker B who believes that if John and Mary are at the party, then so is Bill (i.e., $\Box(Pjm \rightarrow Pb)$). Now, consider the following two utterances made by this speaker:

(3) a. B: John is at the party, or John, Mary and Bill.

$\Box(Pjm \rightarrow Pb)$
$\mathcal{T}_0 = \{{}^{\wedge}Pj, {}^{\wedge}Pm, {}^{\wedge}Pb\}^{\cap} \quad \mathcal{A}_0 = \{{}^{\wedge}Pj, {}^{\wedge}Pjmb\} \qquad p_0 = {}^{\wedge}Pj$

b. (?) B: John is there, or John and Mary, or John, Mary and Bill.

$\Box(Pjm \rightarrow Pb)$
$\mathcal{T}_0 = \{{}^{\wedge}Pj, {}^{\wedge}Pm, {}^{\wedge}Pb\}^{\cap} \quad \mathcal{A}_0 = \{{}^{\wedge}Pj, {}^{\wedge}Pjm, {}^{\wedge}Pjmb\} \qquad p_0 = {}^{\wedge}Pj$

I take it that the utterance in (3b) is intuitively quite strange: why, given B's beliefs, did B include the middle disjunct – or, in terms of attentional intents, why did B intend to draw attention to John and Mary's joint presence (${}^{\wedge}Pjm$)? The maxim of A-Parsimony predicts precisely this strangeness: drawing attention to John and Mary's joint presence is not parsimonious, i.e., superfluous, because B does not consider it possible independently of the presence of all three of them (${}^{\wedge}Pjmb$). More precisely, it can be proven for (3b) that the attentional intent violates

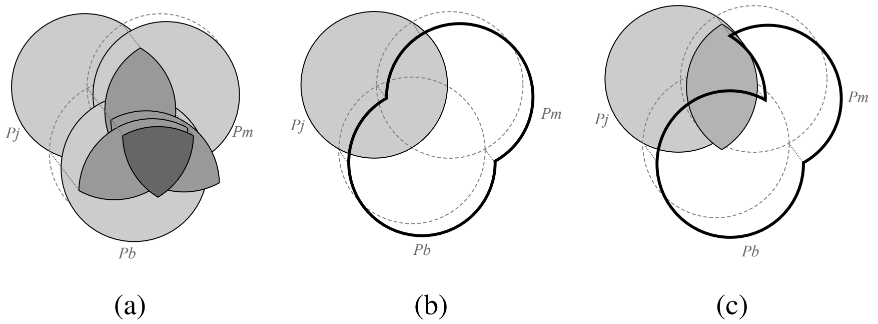


Figure 2: The theme, attentional intents and and exhaustivity implications of (2b) and (2c).

either A-Quality, if $^\wedge Pjmb$ is not in fact considered possible, or A-Parsimony, if it is and $^\wedge Pjm$ is not possible independently of it. In contrast, for (3a) there does exist an admissible model in which all the maxims are complied with.

3. Deriving an exhaustivity operator

I will first apply the maxims to some concrete examples, before presenting a general result and a derivative exhaustivity operator. Consider once again the contrast with which this chapter started, with the assumed themes and intents as given in (2), repeated here:

(2) a. A: Who (of John, Mary and Bill) is at the party?

$\mathcal{T}_0 = \{^\wedge Pj, ^\wedge Pm, ^\wedge Pb\}^\cap \qquad \mathcal{A}_0 = \{^\wedge Pj, ^\wedge Pm, ^\wedge Pb\}^\cap$

b. B: John is at the party.

$\mathcal{T}_0 = \{^\wedge Pj, ^\wedge Pm, ^\wedge Pb\}^\cap \qquad p_0 = {}^\wedge Pj \qquad \mathcal{A}_0 = \{^\wedge Pj\}$

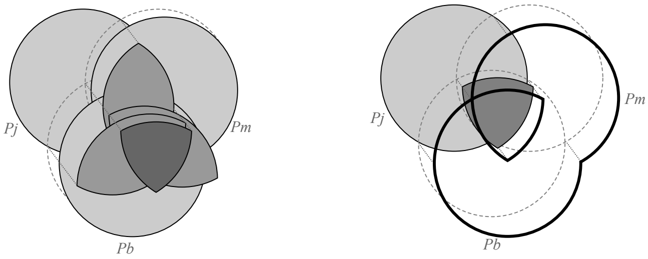
c. B: John is at the party, or John and Mary.

$\mathcal{T}_0 = \{^\wedge Pj, ^\wedge Pm, ^\wedge Pb\}^\cap \qquad p_0 = {}^\wedge Pj \qquad \mathcal{A}_0 = \{^\wedge Pj, ^\wedge Pjm\}$

The exhaustivity implication we wish to derive for (2b) is that B believes that Mary and Bill are not at the party ($\Box\neg Pm$, $\Box\neg Pb$). For (2c) we want to derive the same for Bill ($\Box\neg Pb$), but not for Mary – with regard to Mary we may want to derive merely that the speaker does not consider Mary's presence possible independently of John's ($\Box(Pm \to Pj)$). The desired implications are depicted schematically in figure 2, which is identical to figure 1 given earlier except for the bold outlines, which contain those worlds that the exhaustivity implications would exclude from the speaker's doxastic state. Formally, exhaustivity follows from the assumption that B takes her attentional intent to comply with A-Quantity ($\Box\,\text{A-Q}\textsc{uantity}(\mathcal{A}_0, \mathcal{T}_0)$). That is, for (2b), as depicted in figure 2(b), we get:

Fact 2. For all admissible models $\langle \mathbf{M}, w_0 \rangle$ for (2b):

$$\mathbf{M}, w_0 \models \Box\,\text{A-Q}\textsc{uantity}(\mathcal{A}_0, \mathcal{T}_0) \to (\Box\neg Pb \wedge \Box\neg Pm)$$



(a) (b)

Figure 3: The theme, attentional intent and exhaustivity implication of example (4).

And for (2c), as depicted in figure 2(c), we get:

Fact 3. For all admissible models $\langle \mathbf{M}, w_0 \rangle$ for (2c):

$$\mathbf{M}, w_0 \models \Box\,\text{A-Q}\textsc{uantity}(\mathcal{A}_0, \mathcal{T}_0) \rightarrow (\Box\neg Pb \wedge \Box(Pm \rightarrow Pjm))$$

And there exists such a model where:

$$\mathbf{M}, w_0 \not\models \Box\,\text{A-Q}\textsc{uantity}(\mathcal{A}_0, \mathcal{T}_0) \rightarrow \Box\neg Pm$$

Instead, for all admissible models for (2c) we have:

$$\mathbf{M}, w_0 \models \Box\,\text{A-Q}\textsc{uality}(\mathcal{A}_0) \rightarrow \neg\Box\neg Pm$$

Here the implication that Mary's presence is not possible independently of John's ($\Box(Pm \rightarrow Pjm)$) derives from A-Quantity's conditioning on A-Parsimony: B didn't draw attention to Mary's presence, so she must not consider it possible independently of John and Mary's joint presence. But note that, in this case, the same can be inferred from I-Quality: believing that John is at the party ($\Box Pj$) entails believing that he is if Mary is ($\Box(Pm \rightarrow Pjm)$). Thus, for (2c) A-Parsimony is not strictly necessary. But it does make a difference for the variant in (3a) given earlier, repeated here in (4) (though without the particular beliefs):

(4) B: John is at the party, or John, Mary and Bill.

$\mathcal{T}_0 = \{{}^\wedge Pj, {}^\wedge Pm, {}^\wedge Pb\}^\cap$ $\qquad p_0 = {}^\wedge Pj$ $\qquad \mathcal{A}_0 = \{{}^\wedge Pj, {}^\wedge Pjmb\}$

Figure 3(a) and (b) depict the assumed theme and the attentional intent, in which the bold outline again contains those worlds excluded by the exhaustivity implication from B's doxastic state. The implication is that, according to B, Mary or Bill can be at the party only if everyone is. Unlike before, this implication follows from A-Quantity and not from I-Quality (since $\Box{}^\vee Pj$ does not entail $\Box(Pm \rightarrow Pjmb)$). Hence, A-Parsimony does occasionally make a difference.

To derive a more general result, let us restrict our attention to cases where the theme has the property of *chain completeness*, which means that for every chain of ever more specific, thematic propositions, their infinitary intersection is also thematic. The role of this restriction is clarified in the proof in the appendix. The following result obtains:

Fact 4. For all admissible models **M**, for arbitrary constants $\mathcal{A}_i$ and $\mathcal{T}_j$, where the theme denoted by $\mathcal{T}_j$ is chain-complete:

$$\mathbf{M} \models \Box\text{A-Q}\textsc{uantity}(\mathcal{A}_i, \mathcal{T}_j) \rightarrow \forall a \left(\begin{array}{l} (\mathcal{T}_j(a) \wedge \neg \mathcal{A}_i(a)) \rightarrow \\ \qquad \Box(\neg {}^{\vee}a \vee \exists b(\mathcal{A}_i(b) \wedge (b \subset a) \wedge {}^{\vee}b)) \end{array} \right)$$

And if in **M** the speaker's beliefs are accurate (factivity), then:

$$\mathbf{M} \models \Box\text{A-Q}\textsc{uantity}(\mathcal{A}_i, \mathcal{T}_j) \rightarrow \forall a \left(\begin{array}{l} (\mathcal{T}_j(a) \wedge \neg \mathcal{A}_i(a)) \rightarrow \\ \qquad (\neg {}^{\vee}a \vee \exists b(\mathcal{A}_i(b) \wedge (b \subset a) \wedge {}^{\vee}b)) \end{array} \right)$$

In words: compliance with A-Quantity implies that, for every proposition that is thematic and to which no attention is intended to be drawn, the speaker must think that it does not obtain or that, if it does obtain, a more specific proposition obtains to which attention is intended to be drawn.

We can define a notational shorthand for the second result of fact 4, namely by taking its intension (${}^{\wedge}$), while making sure through abstraction and application that the intent and theme constants are interpreted in the initial world of evaluation. We then get:

Definition 4. For $\mathcal{A}$ and $\mathcal{T}$ any constant or variable of type $\langle\langle s,t\rangle,t\rangle$, let the following notational shorthand be defined:

$$\text{E}\textsc{xh}(\mathcal{A},\mathcal{T}) \stackrel{\text{def}}{=} \lambda \mathcal{T}' \left(\lambda \mathcal{A}' \, {}^{\wedge}\forall a \left(\begin{array}{l} (\mathcal{T}'(a) \wedge \neg \mathcal{A}'(a)) \rightarrow \\ \qquad (\neg {}^{\vee}a \vee \exists b(\mathcal{A}'(b) \wedge (b \subset a) \wedge {}^{\vee}b)) \end{array} \right) (\mathcal{A}) \right) (\mathcal{T})$$

Note that this exhaustivity operator is not a substantive assumption of the theory, but a mere notational shorthand for the exhaustivity implications that are predicted by the theory anyway, at least in admissible models, given factivity and compliance with A-Quantity. The operator can also be formulated in the metalanguage, in a more set-theoretical fashion (parameters $\mathbf{M}, w, g$ for the interpretation function [.], omitted for readability, are the same throughout):

Fact 5. For arbitrary constants or variables $\mathcal{A}$ and $\mathcal{T}$:

$$[\text{E}\textsc{xh}(\mathcal{A},\mathcal{T})] = \bigcap_{\substack{a \in [\mathcal{T}] \\ a \notin [\mathcal{A}]}} (\overline{a} \cup \bigcup_{\substack{b \in [\mathcal{A}] \\ b \subset a}} b)$$

This obtains fairly directly, and I will omit a formal proof: the universal quantifier in definition 4 corresponds here to generalized intersection; negation to complementation, disjunction to union and existential quantification to generalized union. To illustrate, notice that the complements of the bold outlines in figures 2 and 3 given earlier correspond precisely to the sets of worlds characterized by the exhaustivity operator, when applied to the relevant themes and intents.

(a) (b)

Figure 4: The theme and attentional intent (with exhaustivity implication) of (5B).

To illustrate the operator, let us consider a slightly more elaborate example:

(5) A: How many kids does John have?
B: John has one, three, or five kids.

$\mathcal{T}_0 = \{{}^\wedge K0, {}^\wedge K1, {}^\wedge K2, {}^\wedge K3, {}^\wedge K4, \ldots\}$ $\mathcal{A}_0 = \{{}^\wedge K1, {}^\wedge K3, {}^\wedge K5\}$
$p_0 = {}^\wedge K1$ (equivalent to ${}^\wedge(K1 \vee K3 \vee K5)$)

I assume an "at least"-interpretation of numerals for the sake of illustration (the structure of interest can be replicated without numerals). The theme and the attentional intent of (5B) are depicted in figure 4. The striped regions together contain the worlds that are excluded by the operator (like the bold outlines before). The outcome of the operator can be computed as follows, now writing bare numerals n as a shorthand for ${}^\wedge Kn$:

$$\begin{aligned}
\textsc{Exh}(\mathcal{A}_0, \mathcal{T}_0) &= (\overline{0} \cup 1 \cup 3 \cup 5) \cap (\overline{2} \cup 3 \cup 5) \cap (\overline{4} \cup 5) \cap \overline{6} \cap \overline{7} \cap \ldots \\
&= 1 \cap (\overline{2} \cup 3) \cap (\overline{4} \cup 5) \cap \overline{6} \\
&= (1 \cap \overline{2} \cap \overline{4} \cap \overline{6}) \cup \ldots \cup (1 \cap 3 \cap \overline{4} \cap \overline{6}) \cup \ldots \cup (1 \cap 3 \cap 5 \cap \overline{6}) \\
&= (1 \cap \overline{2}) \cup (3 \cap \overline{4}) \cup (5 \cap \overline{6})
\end{aligned}$$

The last line says that John has exactly one, exactly three, or exactly five kids.

4. (Partial) conservativeness with regard to existing operators

The current exhaustivity operator is motivated pragmatically. In contrast, existing operators tend to have been motivated either only descriptively (Groenendijk and Stokhof, 1984; Alonso-Ovalle, 2008) or, in part, in terms of I-Quantity (as in Schulz and Van Rooij 2006; Spector 2007); and sometimes they are conceived of as grammatical devices (e.g., Fox, 2007; Chierchia et al., 2012; Katzir and Singh, 2013). Different motivations may justify different applications of the operator, and hence lead to different empirical predictions even if the operators would be equivalent when regarded purely as abstract mathematical objects. Nevertheless, let us compare the current operator to previous ones at this abstract, mathematical level. I will consider three operators:

1. the *minimal worlds* operator $\textsc{Exh}_{\text{mw}}$ discussed in Spector 2016 (attributed to Spector 2007 and Schulz and Van Rooij 2006);
2. the *innocent exclusion* operator $\textsc{Exh}_{\text{ie}}$ from Alonso-Ovalle 2008 (based on the notion of innocent exclusion from Fox 2007); and
3. the *dynamic* operator $\textsc{Exh}_{\text{dyn}}$ from Schulz and Van Rooij 2006.

I will also discuss the grammatical approach, which applies an operator like $\textsc{Exh}_{\text{mw}}$ to each disjunct separately. Correspondences between these operators and the current one will be stated only for cases in which (i) application of the current operator is warranted, and (ii) the other operator was intended to apply. Condition (i) is satisfied in *operable models*:

Definition 5 (Operable model)**.** An admissible model $\langle \mathbf{M}, w_0 \rangle$ is *operable* if and only if the speaker's beliefs are accurate (factivity), in w_0 the relevant intents comply with the maxims relative to the relevant themes, and the set of thematic propositions in w_0 is chain-complete.

Although I will state (partial) correspondences to each of the aforementioned operators, for reasons of space a proof will be given (in the appendix) only for the third.

First, the **minimal worlds** operator (Schulz and Van Rooij, 2006; Spector, 2007, 2016) can be defined in the current framework by temporarily adding $\textsc{Exh}_{\text{mw}}$ to the language, with the following semantics (as before, the omitted parameters of $[.]$ are the same everywhere):

Definition 6. For arbitrary constants/variables p and $\mathcal{T}$, let:

$$[\textsc{Exh}_{\text{mw}}(p, \mathcal{T})] \stackrel{\text{def}}{=} \{w \in [p] \mid \text{there is no } w' \in [p] \text{ such that: } \{a \mid a \in [\mathcal{T}], w' \in a\} \subset \{a \mid a \in [\mathcal{T}], w \in a\}\}$$

That is, the proposition denoted by p must be true in the relevant worlds w, together with a set of other thematic propositions that is *minimal* compared to the sets of true thematic propositions in other worlds w' in which the proposition denoted by p is true. The minimal worlds operator aligns with the current operator if the attentional intent is a singleton set:

Fact 6. Take any utterance with intents denoted by p_i and $\mathcal{A}_j$ such that $\mathcal{A}_j = \{p_i\}$ is true, and theme denoted by $\mathcal{T}_k$. For any admissible, operable model $\langle \mathbf{M}, w_0 \rangle$ for such an utterance:

$$\mathbf{M}, w_0 \models \textsc{Exh}_{\text{mw}}(p_i, \mathcal{T}_k) = p_i \cap \textsc{Exh}(\mathcal{A}_j, \mathcal{T}_k)$$

This result shows that the operators align when attention does not really make a difference. I refer to Spector 2016 for a detailed comparison of the operator $\textsc{Exh}_{\text{mw}}$ to other existing operators from Krifka 1993 and Fox 2007. None of these operators can distinguish between (1a) and (1b) with which this chapter started – this is the granularity problem.

Second, the **innocent exclusion** operator of Alonso-Ovalle (2008) is formulated in terms of *Alternative Semantics*, but it can be readily applied to attentional intents. It relies on a set IE of *innocently excludable* propositions, a notion adopted from Fox 2007:

Definition 7. For A a set of propositions, and $A^{\cap}$ its closure under intersection, let:

$$\text{IE}(A) \stackrel{\text{def}}{=} \{a \in A^{\cap} \mid \text{for all } b \in A, \text{ any way of excluding from } b \text{ as many } a' \in A^{\cap} \text{ as consistency allows, excludes also } a\}$$

In terms of innocent exclusion, the operator is defined as follows:

Definition 8. For an arbitrary constant/variable $\mathcal{A}$:
$$[\text{EXH}_{\text{ie}}(\mathcal{A})] \stackrel{\text{def}}{=} \bigcup[\mathcal{A}] \cap \bigcap_{a \in \text{IE}([\mathcal{A}])} \overline{a}$$

This operator aligns with the current one with regard to (1a) and (1b) (i.e., (2b,c)). However, for the variant "John, or John, Mary and Bill" in (4) it fails to predict exhaustivity. One problem is that Alonso-Ovalle does not derive the theme from some preceding question, but computes it from the utterance itself by taking the set of disjuncts and closing this set under intersection. A more serious problem is that his operator never excludes *part* of a proposition, like in (4) the part of the proposition denoted by $^\wedge Pjm$ that is not contained in the proposition denoted by $^\wedge Pjmb$ – after all, a proposition is not "innocently excludable" if it contains a proposition that isn't. Still, for a restricted set of cases our operators are formally equivalent:

Fact 7. Take any utterance with intents denoted by p_i and $\mathcal{A}_j$ and theme denoted by $\mathcal{T}_k$ s.t.:

- $p_i = \bigcup \mathcal{A}_j$;
- $\mathcal{T}_k = \mathcal{A}_j^{\cap}$; and
- $\forall a((\mathcal{T}_k(a) \wedge \neg\mathcal{A}_j(a)) \rightarrow \neg\exists b(b \subset a \wedge \mathcal{A}_j(b)))$.

For any admissible, operable model $\langle \mathbf{M}, w_0 \rangle$ for such an utterance:
$$\mathbf{M}, w_0 \models \text{EXH}_{\text{ie}}(\mathcal{A}_j) = (p_i \cap \text{EXH}(\mathcal{A}_j, \mathcal{T}_k))$$

The third bullet excludes cases like (4), where mere parts of propositions need to be excluded.

Third, the **dynamic** operator EXH_{dyn} of Schulz and Van Rooij 2006 is in a way a modification of the "minimal worlds" operator EXH_{mw}, the difference being that EXH_{dyn} does not minimize the set of true thematic propositions among *all* worlds in the informational intent, but only within certain subsets. I will bypass the details of how they determine these subsets, a matter for which they use *dynamic semantics* – in a nutshell, they compare only world-assignment pairs that share the same assignment (discourse referents). At least for disjunctive utterances, which they assume introduce a discourse referent for each disjunct, this amounts simply to comparing only worlds within some proposition in the attentional intent. Hence, for present purposes their operator can be (re)defined as follows:

Definition 9. For arbitrary constants/variables $\mathcal{A}$, and $\mathcal{T}$, let:
$$[\text{EXH}_{\text{dyn}}(\mathcal{A}, \mathcal{T})] = \{w \mid \text{for some } a \in [\mathcal{A}]\text{: } w \in a \text{ and there is no } w' \in a \text{ s.t. } \{b \mid b \in [\mathcal{T}], w' \in b\} \subset \{b \mid b \in [\mathcal{T}], w \in b\}\}$$

Whether this definition corresponds exactly to theirs depends on the degree to which the current attentional intents align with what they consider to be discourse referents – a matter which I defer to Westera 2017. But as it is defined here the operator $\textsc{Exh}_{\text{dyn}}$ can distinguish between (1a) and (1b), as well as account for (4) – indeed, our operators align quite generally:

Fact 8. Take any utterance with intents denoted by p_i and $\mathcal{A}_j$ and a theme denoted by $\mathcal{T}_k$, and for which the following is true:

- $p_i = \bigcup \mathcal{A}_j$; and
- $\mathcal{T}_k = \mathcal{T}_k^{\cap}$.

For any admissible, operable model $\langle \mathbf{M}, w_0 \rangle$ for such an utterance:

$$\mathbf{M}, w_0 \models \textsc{Exh}_{\text{dyn}}(\mathcal{A}_j, \mathcal{T}_k) = (p_i \cap \textsc{Exh}(\mathcal{A}_j, \mathcal{T}_k))$$

This highlights that the contribution of this paper is not the exhaustivity operator in itself but its derivation from Attentional Pragmatics. In contrast, Schulz and Van Rooij offer only a partial explanation for their operator – they do not motivate its sensitivity to discourse referents, which is precisely what gives it an edge over the other operators. The part which they do motivate (basically $\textsc{Exh}_{\text{mw}}$) relies on I-Quantity, and hence runs into the granularity problem.

Lastly, let us consider the **grammatical approach** to exhaustivity, which assumes that operators like $\textsc{Exh}_{\text{mw}}$ are covertly applied to each disjunct separately. The dynamic operator $\textsc{Exh}_{\text{dyn}}$, by virtue of its definition, effectively "exhaustifies" each individual disjunct in a similar fashion. Hence, fact 8 entails a correspondence also between the current operator (as well as $\textsc{Exh}_{\text{dyn}}$) and the grammatical approach. Nevertheless, Attentional Pragmatics *doesn't* predict, unlike the grammatical approach, that individual disjuncts are interpreted exhaustively. Roughly, the difference between our approaches can be paraphrased as follows:

(6) B: John, or John, Mary and Bill.

a. **Grammatical approach:** John *and no one else* was there, or John, Mary and Bill. $(Pj \wedge \neg Pm \wedge \neg Pb) \vee Pjmb)$

b. **Attentional Pragmatics:** John was there, or John, Mary and Bill – *and if Mary or Bill was there then everyone was.* $((Pj \vee Pjmb) \wedge ((Pm \vee Pb) \rightarrow Pjmb))$

Although the two paraphrases (and formulae) are classically, informationally equivalent, only (6a) involves the exhaustive interpretation of an individual disjunct. What this shows is that some at first sight "local" exhaustivity effects can be predicted by a globalist pragmatic theory – of course provided it is sensitive to some dimension of speaker meaning that, while global, reflects the syntactic structure of the uttered sentence more closely than informational intents do (cf. Simons 2011). This doesn't mean that Attentional Pragmatics can account for *all* purportedly local exhaustivity effects – such effects may well be a mixed bag (Geurts, 2011). Nor does a formal correspondence mean that Attentional Pragmatics and the grammatical approach make the same predictions – this depends, after all, on where and when the grammatical approach predicts that operators be inserted, and what their sets of formal alternatives are.

Several proponents of the grammatical approach have proposed that local exhaustification is driven by considerations of redundancy (e.g., Katzir and Singh 2013; essentially following Hurford 1974). They note that, in cases like (6), the stronger disjunct does not contribute to the information conveyed by the utterance *unless* the weaker disjunct is interpreted exhaustively. However, as Ciardelli and Roelofsen (2016) demonstrate, this explanation may fail when one's semantics/pragmatics is more fine-grained than the traditional, information-only picture: for instance, constituents that are informationally redundant can still make an attentional difference. Indeed, according to Attentional Pragmatics the second disjunct in (6) isn't redundant, and as a consequence this theory is incompatible with redundancy-based accounts of local exhaustification in such disjunctions. Note that this doesn't mean that redundancy has no role to play in pragmatics. For one, informational redundancy may still play a role in *con*junctions where one conjunct entails the other (because from I-Quality and A-Parsimony it follows that the attentional intent may rationally contain only the conjunction as a whole). Moreover, the maxim of A-Parsimony essentially bans an *attentional* kind of redundancy.

5. Discussion

According to Attentional Pragmatics, exhaustivity implications arise not from the assumption that a rational speaker asserts all thematic propositions believed to be true, nor from local exhaustification driven by considerations of redundancy, but from the assumption that a rational speaker draws attention to all thematic propositions believed to be possible. This new perspective provides a globalist pragmatic solution to the granularity problem, one which the current paper captured in an operator. It also solves several other problems for the standard, information-based approach; I refer to Westera 2017 for an application to the occurrence of exhaustivity in cases where I-Quantity does not apply (hints, questions) and cases where the opinionatedness assumption is explicitly denied, as well as a solution to the symmetry problem. If valid, this attentional approach mandates a thorough revision of the literature on exhaustivity.

At a purely technical level, the current paper may seem to suggest that the required revision is rather minimal: the operator derived from Attentional Pragmatics is, as a purely formal device, in important respects conservative with regard to operators from the literature. However, the range of circumstances in which its application is pragmatically warranted is more restricted than the frequent reliance on exhaustivity operators in the literature seems to require. It excludes, for instance, the local operators that grammaticalists assume, but also certain global occurrences. For instance, Schulz and Van Rooij (2006) seek to apply their operator directly to conditional answers to unconditional questions, and to modalized answers to non-modalized questions, but these are cases for which in the current approach no "operable" admissible models exist: conditional/modal answers cannot compliantly address the theme introduced by unconditional/non-modal questions, hence we must not seek to directly apply our operator to them. These cases must be analyzed, rather, as involving a *theme shift*, to be explained by a separate "theme pragmatics" (Westera, 2017), i.e., a theory of how conversational goals are prioritized and organized into themes. The current maxims, in contrast, constrain only which intentions are rational *given* a theme, i.e., a set of goals.

More generally, the restrictions on the current operator's applicability reflect that it abbreviates only a rather small part of a pragmatic theory. The operator of Schulz and Van Rooij seems more ambitious; they motivate it by stating that "none of these [previously proposed] theories gives a satisfying explanation for why the scope of exhaustive interpretation should be restricted to those cases that they can actually handle" (p.8). But the converse is true as well: none of these existing theories, including Schulz and Van Rooij's, gives a satisfying explanation for why the cases that their operators appear to handle are cases that they *should* handle. If all you have is a hammer, everything looks like a nail (e.g., Maslow, 1966). By acknowledging the limited applicability of the current exhaustivity operator, and the unknown applicability of existing ones, we may begin to see subtle differences between different types of exhaustivity-like inferences, and between the types of conversational goals that are normally served by conditional and unconditional answers or plain and modalized answers.

Appendix. Proofs of facts 4 and 8

Proof of fact 4: Take an arbitrary admissible model $\mathbf{M}$ in which $\mathcal{T}_j$ complies with the chain completeness restriction. Take an arbitrary world w in this model. Suppose that the speaker takes A-Quantity to be complied with, i.e., $\mathbf{M}, w \models \Box\,\text{A-Q}\textsc{uantity}(\mathcal{A}_i, \mathcal{T}_j)$. Given intent and theme introspection, this means that A-Quantity is actually complied with in w:

$$\mathbf{M}, w \models \forall a \left(\begin{pmatrix} \text{A-Q}\textsc{uality}(\{a\}) \wedge \\ \text{A-R}\textsc{elation}(\{a\}, \mathcal{T}_j) \wedge \\ \text{A-P}\textsc{arsimony}(\{a\}, \mathcal{T}_j) \end{pmatrix} \rightarrow \mathcal{A}_i(a) \right) \tag{1}$$

Take an arbitrary function g that assigns to a a thematic proposition, i.e., suppose that:

$$\mathbf{M}, w, g \models \mathcal{T}_j(a) \tag{2}$$

Suppose, furthermore, that no attention is drawn to a in w:

$$\mathbf{M}, w, g \models \neg \mathcal{A}_i(a) \tag{3}$$

Since $\mathcal{A}_i(a)$ is false in w, the antecedent in (1) cannot be true either, hence at least one of its conjuncts must be false. A-Relation cannot be blamed, because the proposition denoted by a is thematic in w (from supposition (2)), so it must be either A-Quality or A-Parsimony. Let us explore the latter.

Suppose that the singleton intent denoted by $\{a\}$ does not comply with A-Parsimony in w, i.e., $\mathbf{M}, w, g \models \neg\,\text{A-P}\textsc{arsimony}(\{a\}, \mathcal{T}_0)$. This amounts to:

$$\mathbf{M}, w, g \models \Diamond^{\vee} a \wedge \Box({}^{\vee} a \rightarrow \exists b\,(b \subset a \wedge \mathcal{T}_0(b) \wedge {}^{\vee} b)) \tag{4}$$

It follows that there exists a world w' that is belief-accessible from w, such that the proposition assigned to a is true in w', and, by the second conjunct, some stronger proposition can be assigned to b that is true and thematic in w'. This means that in the original world w, the proposition assigned to b must be considered possible and, by theme introspection, thematic. Hence, we have:

$$\mathbf{M}, w, g \models \exists b (b \subset a \wedge \mathcal{T}_0(b) \wedge \Diamond^{\vee} b)$$

Since this stronger proposition b is thematic and possible, A-Quantity (which is complied with according to (1)) requires that it be an element of the attentional intent denoted by $\mathcal{A}_i$ in w, unless A-Parsimony prevents this, i.e., unless there is an even stronger thematic and possible proposition (say, c), independently of which $\mathscr{b}$ in turn is not considered possible. And so on, potentially *ad infinitum*.

To curb this potential infinitude, assume that the set of thematic propositions is *chain-complete*, i.e., that for every chain of increasingly specific thematic propositions $a_0, a_1, \ldots$ (i.e., such that every $a_{i+1} \subset a_i$), their infinitary intersection $\bigcap\{a_0, a_1, \ldots\}$ is also thematic. This guarantees that there exists a maximally specific thematic and possible proposition, and according to A-Quantity that must be an element of the attentional intent denoted by $\mathcal{A}_i$. This means that we can strengthen supposition (4) by adding the conjunct $\mathcal{A}_i(\mathscr{b})$, which after dropping the conjunct $\mathcal{T}_0(\mathscr{b})$ yields the following:

$$\mathbf{M}, w, g \models \Diamond^{\vee}a \wedge \Box(^{\vee}a \rightarrow \exists \mathscr{b}\,(\mathscr{b} \subset a \wedge \mathcal{A}_i(\mathscr{b}) \wedge {}^{\vee}\mathscr{b}))$$

This was derived, recall, from the supposition that the singleton intent denoted by $\{a\}$ does not comply with A-Parsimony in w, i.e., that A-Parsimony is the reason why the proposition assigned to a is not an element of the attentional intent denoted by $\mathcal{A}_i$. The other possible reason was A-Quality, i.e., $\neg\Diamond^{\vee}a$. Hence, we can conclude the disjunction of these two reasons:

$$\mathbf{M}, w, g \models \neg\Diamond^{\vee}a \vee (\Diamond^{\vee}a \wedge \Box(^{\vee}a \rightarrow \exists \mathscr{b}\,(\mathscr{b} \subset a \wedge \mathcal{A}_i(\mathscr{b}) \wedge {}^{\vee}\mathscr{b})))$$

which implies: $\mathbf{M}, w, g \models \Box(\neg^{\vee}a \vee \exists \mathscr{b}\,(\mathscr{b} \subset a \wedge \mathcal{A}_i(\mathscr{b}) \wedge {}^{\vee}\mathscr{b}))$

By retracting suppositions (2) and (3) we obtain:

$$\mathbf{M}, w \models \forall a \begin{pmatrix} (\mathcal{T}_j(a) \wedge \neg\mathcal{A}_i(a)) \rightarrow \\ \qquad \Box(\neg^{\vee}a \vee \exists \mathscr{b}(\mathcal{A}_i(\mathscr{b}) \wedge \mathscr{b} \subset a \wedge {}^{\vee}\mathscr{b})) \end{pmatrix}$$

And by retracting supposition (1), i.e., that A-Quantity is (believed to be) complied with, we obtain the first result in fact 4. The second result directly derives from this through factivity.

The restriction to chain-complete themes is only a presentational choice, that allows a simpler formulation of the main result. It is not indicative of, say, some sort of defect in the maxims. I don't think that, relative to a theme like that is not chain-complete, a speaker could rationally behave differently from what the current maxims predict, namely, to not draw attention to any particular proposition in the chain. If anything, a rational speaker may want to consider switching to a chain-complete theme instead.

Proof of fact 8: We prove the equivalence of the two operators by proving inclusion right-to-left and then left-to-right. First right-to-left: in an arbitrary admissible, operable model $\langle \mathbf{M}, w_0 \rangle$ of the specified type of utterance, take a world $w \in [p_i \cap \text{EXH}(\mathcal{A}_j, \mathcal{T}_k)]_{\mathbf{M}, w_0, g}$. Given that $p_i = \bigcup \mathcal{A}_j$ is true in w_0, there must be some $a \in [\mathcal{A}_j]_{\mathbf{M}, w_0, g}$ such that $w \in a$. Moreover, given the chain-completeness restriction on themes in operable models, and given compliance with A-Relation, there must be a most specific (strongest, smallest) proposition a of that sort. From our exhaustivity operator it follows that every thematic proposition to which no attention is drawn is either false in w, or entailed by this most specific a. Hence, w makes the proposition a true and anything entailed by it, but no other thematic propositions. Within a, then, there is no $w' \in a$ where the set of true thematic propositions is smaller than in w. Hence (by definition) $w \in [\text{EXH}_{\text{dyn}}(\mathcal{A}_j, \mathcal{T}_k)]_{\mathbf{M}, w_0, g}$.

Conversely, take an arbitrary world $w \in [\textsc{Exh}_{\text{dyn}}(\mathcal{A}_j, \mathcal{T}_k)]_{\mathbf{M},w_0,g}$. According to the definition of $\textsc{Exh}_{\text{dyn}}$, there must be some $a \in [\mathcal{A}_j]_{\mathbf{M},w_0,g}$ such that $w \in a$ and w makes a minimal number of thematic propositions true, compared to other $w' \in a$. Given the chain-completeness restriction and compliance with A-Relation, there must be a most specific (strongest, smallest) a of that sort. Within this most specific a, any minimal set of true thematic propositions will contain a and anything entailed by it, but nothing else. (This is because, if a minimal set of true thematic propositions had contained another thematic proposition a', then the intersection $a \cap a'$ would have been thematic as well (by assumption of closure under intersection), and a would not have been possible independently of these more specific intersections, contrary to A-Parsimony, and would not have been included in the attentional intent.) Hence, this world w is contained in a, to which attention is drawn, but in no more specific thematic proposition. By definition, my operator contains all such worlds. Moreover, given that $p_i = \bigcup \mathcal{A}_j$ is true in w_0, we have that $w \in [p_i]_{\mathbf{M},w_0,g}$, and hence $w \in [p_i \cap \textsc{Exh}(\mathcal{A}_j, \mathcal{T}_k)]_{\mathbf{M},w_0,g}$.

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