An attention-based explanation for some exhaustivity operators

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The standard pragmatic recipe for exhaustivity implications is based on Grice's maxim of Quantity plus a competence assumption. In recent years a number of serious problems for this recipe have been (re)discovered. I present a formal, unifying solution that, moreover, generates existing exhaustivity operators from more basic, pragmatic assumptions.

1. Four problems for the standard recipe

- **Granularity:** Sentences that are classically (informationally) equivalent may nevertheless differ in exhaustivity implications (Van Rooij & Schulz, 2006; Katzir & Singh, 2013):

  (1) Q: Who (of John, Mary and Bill) was at the party?
  a. A: John. *(Exh.: Mary and Bill weren't.)*
  b. A: John, or both John and Mary. *(Exh.: Bill wasn't.)*
  c. A: John, or everyone. *(Exh.: if Mary or Bill, then everyone.)*

- **Questions:** Questions arguably lack an informational intent for Quantity to apply to, but they nevertheless imply exhaustivity (e.g., Biezma & Rawlins, 2012):

  (2) Was John there, or Mary? *(Exh.: not both)*

  (Note that what is *implied* is not necessarily *implicated*, and may well be *presupposed*.)

- **Hints:** Hints are arguably exempt from Quantity, but do imply exhaustivity (Fox, 2014):

  (3) There is money in box 20 or 25. *(Exh.: not both)*

  See Fox for a rebuttal of a number of possible coping strategies for the standard recipe.

- **Competence:** Exhaustivity occurs without a competence assumption (Westera, 2013b):

  (4) Q: You may not know this, but who (of John, Mary and Bill) was at the party?
  A: John and Mary were there. *(Exh.: Bill wasn't)*

  See (Westera, 2013b) for an argument that purported evidence in favour of the reliance on a competence assumption (e.g., Breheny et al. 2013) has been misinterpreted. (I will set aside the important role of *intonation*, favouring an account according to which a final fall conveys compliance with the maxims; Westera, 2013a.)

A fifth problem that the current approach will solve is the *symmetry problem*, but space does not permit a discussion. A sixth problem, to which I will not propose a (general) solution, is that of *embedded* exhaustivity. I consider a globalist solution to the above problems worth pursuing regardless of whether embedded and unembedded cases are a single phenomenon.

2. A new recipe based on attention

I adopt the common assumption that a disjunction evokes its disjuncts as *alternatives* (e.g., Aloni, 2007), conceived of as possibilities to which a speaker intends to *draw attention* (Ciardelli et al. 2009). As a communicative intention this must be governed by an appropriate set of maxims. I define these in Montague's *Intensional Logic* (IL), with doxastic speaker modalities ($\Box$, $\Diamond$) and set-theoretical shorthands. Let $\boldsymbol{a}, \boldsymbol{b}$,... be constants/variables of type $\langle s, t\rangle$ (propositions) and $\mathcal{A}, \mathcal{B}$,... of type $\langle\langle s, t\rangle, t\rangle$ (sets of propositions). To illustrate, let us first define several of Grice's maxims, or *I(nformation)-maxims.* Given an informational intent $\boldsymbol{p}$ and a question under discussion (Qud) $\mathcal{Q}$:

$\text{I-Quality}(p) = \Box{}^{\vee}p$ $\qquad$ $\text{I-Relation}(\mathcal{Q}, p) = \mathcal{Q}(p)$
$\text{I-Quantity}(\mathcal{Q}, p) = \forall q\,\big(\big(\text{I-Quality}(q) \wedge \text{I-Relation}(\mathcal{Q}, q)\big) \rightarrow (p \subseteq q)\big)$

In words: intend to share only information that you believe is true (I-Quality) and that answers the QUD (I-Relation); and share *all* such information (I-Quantity). (I will ignore *indirect* answers, which would require a more sophisticated I-Relation and I-Quantity.)

For an *attentional* intent $\mathcal{A}$ (evoked alternatives) let us define similar maxims:

$\text{A-Quality}(\mathcal{A}) = \forall a\,(\mathcal{A}(a) \rightarrow \Diamond({}^{\vee}a \wedge \forall b\,((\mathcal{Q}(b) \wedge b \subset a) \rightarrow \neg{}^{\vee}b)))$
$\text{A-Relation}(\mathcal{Q}, \mathcal{A}) = \forall a(\mathcal{A}(a) \rightarrow \mathcal{Q}(a))$
$\text{A-Quantity}(\mathcal{Q}, \mathcal{A}) = \forall a((\text{A-Quality}(\{a\}) \wedge \text{A-Relation}(\mathcal{Q}, \{a\})) \rightarrow \mathcal{A}(a))$

In words: intend to draw attention only to answers to the QUD (A-Relation) that you consider possible independently of any stronger answers (A-Quality); and draw attention to *all* of those (A-Quantity). I intend the independence requirement in A-Quality to reflect considerations of attentional economy.

Let an *admissible model* be a model for IL in which the foregoing definitions (grey boxes) are valid (cf. meaning postulates for Montague), the denotations of $\mathcal{Q}$ are closed under (infinitary) intersection, and the set of worlds is sufficiently large to distinguish all contingent first-order formulae (needed for Fact 2). It can be proven that compliance with A-Quantity means that every answer to the QUD to which no attention is drawn, is believed to be either false or only ever true together with some stronger answer to which attention is drawn:

Fact 1. In all admissible models $\mathbf{M}$, for arbitrary constants $\mathcal{A}, \mathcal{Q}$:
$\mathbf{M} \models \text{A-Quantity}(\mathcal{Q}, \mathcal{A}) \rightarrow \forall a\,((\mathcal{Q}(a) \wedge \neg\mathcal{A}(a)) \rightarrow \Box(\neg{}^{\vee}a \vee \exists b(\mathcal{A}(b) \wedge (b \subset a) \wedge {}^{\vee}b)))$

3. Solutions to the four problems

- **Granularity**: For (1a,b,c), let $\mathcal{Q}$ denote the closure of $\{{}^{\wedge}Pj, {}^{\wedge}Pm, {}^{\wedge}Pb\}$ under $\cap$ and $\cup$ (here Pj translates "John was at the party", etc.). Let $\mathcal{A}$ denote the sets $\{{}^{\wedge}Pj\}$ for (1a), $\{{}^{\wedge}Pj, {}^{\wedge}(Pj \wedge Pm)\}$ for (1b) and $\{{}^{\wedge}Pj, {}^{\wedge}(Pj \wedge Pm \wedge Pb)\}$ for (1c). The predictions are as desired; for reasons of space I will state the formal result only for (1c):

  Fact 2. For all admissible models $\mathbf{M}$ for (1c), with $\mathcal{A}$ and $\mathcal{Q}$ as just explained:
  $\mathbf{M} \models \text{A-Quantity}(\mathcal{Q}, \mathcal{A}) \rightarrow (\neg Pm \vee (Pj \wedge Pm \wedge Pb)) \wedge (\neg Pb \vee (Pj \wedge Pm \wedge Pb))$

- **Questions:** For (2), take $\mathcal{Q} = \{{}^{\wedge}Pj, {}^{\wedge}Pm, {}^{\wedge}(Pj \wedge Pm)\}$ and $\mathcal{A} = \{{}^{\wedge}Pj, {}^{\wedge}Pm\}$. Although I-Quantity does not apply to questions, A-Quantity does, and delivers the right result.
- **Hints:** This is solved only conceptually: I propose that (3) is exempt from I-Quantity but not A-Quantity: while withholding information is part of their job, a quizmaster who doesn't draw attention to all live options is guilty of misleading (and should be fired).
- **Competence:** Exhaustivity was derived in Facts 1 & 2 without a competence assumption. This is because A-Quantity is in a relevant respect more demanding than I-Quantity, i.e., its antecedent is weaker ($\Diamond$ instead of $\Box$) and hence the negation of its antecedent is sufficiently strong for exhaustivity.

4. Formal comparison

For easy comparison let us capture the consequent of fact 1, strengthened by factivity (that the beliefs are true), in an 'exhaustivity operator':

$$\text{EXH}(\mathcal{Q},\mathcal{A}) \overset{\text{def}}{=} {}^\wedge\forall a \begin{pmatrix} (\mathcal{Q}(a) \wedge \neg\mathcal{A}(a)) \rightarrow \\ (\neg{}^\vee a \vee \exists b(\mathcal{A}(b) \wedge (b \subset a) \wedge {}^\vee b)) \end{pmatrix} \text{ (equivalent to } \bigcap_{\substack{a\in\mathcal{Q}\\ a\notin\mathcal{A}}} (\overline{a} \cup \bigcup_{\substack{b\in\mathcal{A}\\ b\subset a}} b))$$

I intend this merely as a notational shorthand, not as a grammatical operator. Now it can be proven that (a) with information only, my EXH is conservative with regard to *predicate minimization* EXH_{mw} as discussed (a.o.) in Spector, 2016; and (b) the A-maxims generate the more advanced, *dynamic* operator EXH_{dyn} of Van Rooij and Schulz, who in fact explain only EXH_{mw} (in terms of the standard recipe) and just stipulate EXH_{dyn}:

Fact 4. For all admissible $\mathbf{M}$, if p and $\mathcal{A}$ comply with the I/A-maxims relative to $\mathcal{Q}$:

a. if $\mathcal{A}$ denotes the singleton set $\{p\}$:
$\mathbf{M} \models p \cap \text{EXH}(\mathcal{Q},\mathcal{A}) = \text{EXH}_{\text{mw}}(\mathcal{Q}, p)$

b. if for some predicate P and domain D, $\mathcal{Q} = \{{}^\wedge Px \mid x \in D\}$ closed under $\cap$, and for C a context-change potential that provides information $\bigcup\mathcal{A}$ and assigns all and only $a \in \mathcal{A}$ to a discourse referent:
$\mathbf{M} \models p \cap \text{EXH}(\mathcal{Q},\mathcal{A}) = \text{EXH}_{\text{dyn}}(P, C)$

This correspondence notwithstanding, if p or $\mathcal{A}$ violates a maxim relative to $\mathcal{Q}$, our operator may deliver nonsense, and others remain unexplained.

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