Modified numerals in inquisitive pragmatics

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Goal

To defend a simple and uniform semantic analysis of (modified) numerals in light of data suggesting the contrary.
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4 boys came  \[ \exists x. |x| = 4 \land B(x) \land C(x) \]
At least 3 boys came  \[ \exists x. |x| \geq 3 \land B(x) \land C(x) \]
More than 2 boys came  \[ \exists x. |x| > 2 \land B(x) \land C(x) \]
At most 5 boys came  \[ \exists x. |x| \leq 5 \land B(x) \land C(x) \]
Fewer than 6 boys came  \[ \exists x. |x| < 6 \land B(x) \land C(x) \]
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Some facts

- At least \( n \) boys came \( \equiv \) More than \( n - 1 \) boys came  
  \( (\equiv_T n \text{ boys came}) \)
- At most \( n \) boys came \( \equiv \) Fewer than \( n + 1 \) boys came
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- The sentences with DE modifiers are always true.
Puzzle 1: ‘n’ vs. ‘at least n’

(1) a. 3 boys came. \(\sim\) Exactly 3 boys came.
b. At least 3 boys came. \(\not\sim\) Exactly 3 boys came.
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Explanation:

- a. and b. are truth-conditionally equivalent, but nevertheless semantically distinct.
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   - The **exhaustivity** inference of a. is a pragmatic implicature stemming from this semantic distinction.
Puzzle 2: ‘At most’ and ‘fewer than’

The following is falsely predicted to be always true:

(2) \{At most/fewer than\} 3 boys came
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- Somehow, for (2), this implicature is much more typical, perhaps even always there.
Puzzle 3: Unmodified vs. modified

Modified numerals enable anaphora only to the ‘maximal set’, unmodified numerals also to the ‘witness’ set:
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- Perhaps b. by default implicates exhaustivity in some as yet undiscovered way.
Puzzle 4: Comparative vs. superlative modifiers (1)
Nouwen (2010)

[Knowing that a hexagon has exactly six sides]

(4) A hexagon has \[
\begin{cases}
\text{at least 5} \\
\text{more than 4} \\
\text{at most 7} \\
\text{fewer than 8}
\end{cases}
\] sides.

Explanation:
- Only superlative modifiers convey ignorance (Nouwen)
- Only superlative modifiers convey possibility (Nouwen)

More precisely:
- The relevant inferences are pragmatic implicatures.
- Comparative modifiers are used with a singleton domain restriction ('referentially') more easily than superlative modifiers, in which case the implicatures are absent.

Prediction: 'At least/at most 6' are perhaps better.
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Puzzle 4: Comparative vs. superlative (2)
Geurts, et al. (2010)

Argument validity judgements:

<table>
<thead>
<tr>
<th></th>
<th>Berta had 3 beers</th>
<th>Berta had at least 3 beers</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Berta had 3 beers</td>
<td>Berta had more than 2 beers</td>
<td>100</td>
</tr>
<tr>
<td>b</td>
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<td>Berta had at most 3 beers</td>
<td>61</td>
</tr>
<tr>
<td>c</td>
<td>Berta had 3 beers</td>
<td>Berta had fewer than 4 beers</td>
<td>93</td>
</tr>
<tr>
<td>d</td>
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Explanation:

- a and c are blocked by the ignorance conveyed by their conclusion (Geurts, et al.)
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Explanation:

- a and c are blocked by the **ignorance** conveyed by their conclusion (Geurts, et al.)
- (But that does not mean the ignorance is a semantic entailment (Coppock and Brochhagen (submitted)))
Puzzle 5: ‘At most’ vs. the rest
Coppock and Brochhagen (submitted)

[Picture of four apples on a table] Truth judgment:

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\begin{align*}
(5) \quad \{ & \text{At least 3} \\
& \text{More than 2} \\
& \text{At most 5} \\
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apples are on the table.
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Explanation:

- This setting disables \textit{ignorance} inferences, for some reason.
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- However, it does not disable \textit{possibility} inferences, for some reason.
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Explanation:

- This setting disables ignorance inferences, for some reason.
- However, it does not disable possibility inferences, for some reason.
- In this case, for ‘at least 3’ the possibility inference happens to be true, for ‘at most 5’ it is false.
Hypothesis

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Shopping list:
- \textbf{Ignorance}: I’m not sure how many exactly.
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Shopping list:
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- **Exhaustivity’**: All boys who came to the party wore a hat.
Structure

Framework

Solving the puzzles

Conclusion
Inquisitive semantics

Unrestricted inquisitive semantics

1. \([P(t_1, \ldots, t_n)]_g = \{w | ([t_1]_{w,g}, \ldots, [t_n]_{w,g}) \in [P]_w\}\)

2. \([\varphi \lor \psi]_g = [\varphi]_g \cup [\psi]_g\)

3. \([\varphi \land \psi]_g = [\varphi]_g \cap [\psi]_g\) (where \(A \cap B = \{\alpha \cap \beta : \alpha \in A, \beta \in B\}\))

4. \([\exists x.\varphi]_g = \bigcup_{d \in D} [\varphi]_g[x/d]\)

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Entailment

1. \( A \) entails \( B \), \( A \models B \), iff \( \exists C, B \cap C = A \)
2. \( A \) contains \( B \), \( B \subseteq A \), iff \( \exists C, B \cup C = A \)
Inquisitive pragmatics
(Grice, 1975)

Maxim of Relation
Only propose what is relevant.

Maxim of Quantity
Make your contribution just as informative as required for the current goal of the conversation.

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Propose a proposition only if you believe it to be true.
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- Goal (ex.): give as many answers to the $QUD$ as needed.
Two additional background assumptions

Focus Principle (Beaver and Clark, 2008; Rooth, 1996)

A focused constituent presupposes a QUD to which it is an answer.
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Consequences:
  ▶ Modifiers ‘at least’ etc. presuppose a QUD to which their prejacent is an answer (Beaver and Coppock, 2012).

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A quantifier may have an implicit, possibly singleton domain restriction that is known to only the speaker.
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- (1) presupposes the QUD ‘how many boys came?’.
- In light of this QUD, (2) has an exhaustivity implicature.
- Perhaps then the implicature is lexicalized (but this makes no difference).
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- (3b) never implicates exhaustivity, however, its responses might.
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Modified numerals enable anaphora only to the ‘maximal set’, unmodified numerals also to the ‘witness’ set:

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They all wore a hat. \(\therefore\) All boys who came wore hats.  
b. \{At least/exactly/fewer than\} 3 boys came to the party.  
They all wore a hat. \(\therefore\) All boys who came wore hats.

More precisely:

- For (3a), the exhaustivity implicature and, with it, the maximal set anaphora, is optional.
- (3b) never implicates exhaustivity, however, its responses might.
- Any response to (3b) that reveals the contents of the discourse referent, will implicate exhaustivity.
Puzzle 3: Unmodified vs. modified

Modified numerals enable anaphora only to the ‘maximal set’, unmodified numerals also to the ‘witness’ set:

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   They all wore a hat. \sim / \n\sim All boys who came wore hats.

b. \{At least/exactly/fewer than\} 3 boys came to the party.
   They all wore a hat. \sim All boys who came wore hats.

More precisely:

- For (3a), the exhaustivity implicature and, with it, the maximal set anaphora, is optional.
- (3b) never implicates exhaustivity, however, its responses might.
- Any response to (3b) that reveals the contents of the discourse referent, will implicate exhaustivity.

**Prediction:** For ‘3’, ‘some’ and ‘many’, the kind of anaphora is QUD-dependent.
Puzzle 4: Comparative vs. superlative modifiers (1)
Nouwen (2010)

[Knowing that a hexagon has exactly six sides]

\[
\begin{align*}
\text{(4) A hexagon has } & \begin{cases} 
\text{\#at least 5} \\
\text{more than 4} \\
\text{\#at most 7} \\
\text{fewer than 8} 
\end{cases} 
\text{ sides.}
\end{align*}
\]

Explanation:
- Only superlative modifiers convey \textit{ignorance} (Nouwen)
- Only superlative modifiers convey \textit{possibility} (Nouwen)
Puzzle 4: Comparative vs. superlative modifiers (1)
Nouwen (2010)

[Knowing that a hexagon has exactly six sides]

(4) A hexagon has \left\{ \begin{align*}
& \# \text{at least 5} \\
& \text{more than 4} \\
& \# \text{at most 7} \\
& \text{fewer than 8}
\end{align*} \right\} \text{ sides.}

Explanation:
- Only superlative modifiers convey \textbf{ignorance} (Nouwen)
- Only superlative modifiers convey \textbf{possibility} (Nouwen)

More precisely:
- The relevant inferences are pragmatic implicatures.
Puzzle 4: Comparative vs. superlative modifiers (1)
Nouwen (2010)

[Knowing that a hexagon has exactly six sides]

\[
\begin{align*}
\text{(#at least 5 \ more than 4 \ #at most 7 \ fewer than 8)}
\end{align*}
\]

(4) A hexagon has \[\{\text{#at least 5 \ more than 4 \ #at most 7 \ fewer than 8}\}\] sides.

Explanation:

- Only superlative modifiers convey \textit{ignorance} (Nouwen)
- Only superlative modifiers convey \textit{possibility} (Nouwen)

More precisely:

- The relevant inferences are pragmatic implicatures.
- Comparative modifiers are used with a singleton domain restriction (‘referentially’) more easily than superlative modifiers, in which case the implicatures are absent.
Puzzle 4: Comparative vs. superlative modifiers (1)
Nouwen (2010)

[Knowing that a hexagon has exactly six sides]

(4) A hexagon has \[
\begin{cases}
\#\text{at least 5} \\
\#\text{more than 4} \\
\#\text{at most 7} \\
\#\text{fewer than 8}
\end{cases}
\]
sides.

Explanation:
- Only superlative modifiers convey ignorance (Nouwen)
- Only superlative modifiers convey possibility (Nouwen)

More precisely:
- The relevant inferences are pragmatic implicatures.
- Comparative modifiers are used with a singleton domain restriction (‘referentially’) more easily than superlative modifiers, in which case the implicatures are absent.
- **Prediction**: ‘At least/at most 6’ are perhaps better.
Puzzle 5: ‘At most’ vs. the rest
Coppock and Brochhagen (submitted)

[Picture of four apples on a table] Truth judgment:

(5) \[
\begin{align*}
\text{At least 3} \\
\text{More than 2} \\
?\text{At most 5} \\
\text{Fewer than 6}
\end{align*}
\]

apples are on the table.

Explanation:

- This setting disables **ignorance** inferences, for some reason.
- However, it does not disable **possibility** inferences, for some reason.
- In this case, for ‘at least 3’ the possibility inference happens to be true, for ‘at most 5’ it is false.
Structure

Framework

Solving the puzzles

Conclusion
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  - The uniform semantics can be maintained.
  - Each of the distinguishing inferences has a pragmatic origin, not semantic.
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- Together with the Focus Principle and the Privacy Principle, all contrasts were accounted for.
Conclusion

- I have tried to defend the hypothesis that:
  - The uniform semantics can be maintained.
  - Each of the distinguishing inferences has a pragmatic origin, not semantic.
- Inquisitive semantics and pragmatics is a concise but powerful toolbox.
- Together with the Focus Principle and the Privacy Principle, all contrasts were accounted for.
- This highlights the importance of taking into account implicit QUDs when doing linguistic experiments.
Thanks!

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