Meanings as Proposals: 
an Inquisitive Approach to Exhaustivity

Matthijs Westera

Institute for Logic, Language and Computation
University of Amsterdam

Genève, ICL, July 26th 2013
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Goal of this talk

(1) Which colours (among red, green and blue) does John like? He likes blue. \(\sim He \ doesn't \ like \ red, \ green.\)
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   He likes blue. \(\sim He\ doesn't\ like\ red,\ green.\)

Conversational implicature (Grice, 1975)
An implicature, the supposition of which is necessary for maintaining the assumption that the speaker is cooperative.
Goal of this talk

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Conversational implicature (Grice, 1975)

An implicature, the supposition of which is necessary for maintaining the assumption that the speaker is cooperative.

1. Had sp. believed John likes red, she should have said so.
Goal of this talk

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Conversational implicature (Grice, 1975)
An implicature, the supposition of which is necessary for maintaining the assumption that the speaker is cooperative.

1. Had sp. believed John likes red, she should have said so.
2. She doesn’t believe that he likes red.
Goal of this talk

(1) Which colours (among red, green and blue) does John like?
He likes blue. \(\sim He\ doesn't\ like\ red,\ green.\)

Conversational implicature (Grice, 1975)
An implicature, the supposition of which is necessary for maintaining the assumption that the speaker is cooperative.

1. Had sp. believed John likes red, she should have said so.
2. She doesn’t believe that he likes red.
   
   ... 

3. She believes that he doesn’t like red.
Goal of this talk

(1) Which colours (among red, green and blue) does John like?
   He likes blue.  \[\sim He \ doesn't \ like \ red, \ green.\]

Conversational implicature (Grice, 1975)
An implicature, the supposition of which is necessary for maintaining the assumption that the speaker is cooperative.

1. Had sp. believed John likes red, she should have said so.
2. She doesn’t believe that he likes red.
   \[\ldots \ (\text{'the epistemic step'} - \text{Sauerland, 2004})\]
3. She believes that he doesn’t like red.
Goal of this talk

(1) Which colours (among red, green and blue) does John like?
He likes blue. \( \sim He \, doesn't \, like \, red, \, green. \)

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3. She believes that he doesn’t like red.

“[the epistemic] step does not follow from Gricean maxims and logic alone.” - Chierchia, et al. (2008)
Goal of this talk

(1) Which colours (among red, green and blue) does John like?
He likes blue.  \( \Rightarrow \) He doesn’t like red, green.

Conversational implicature (Grice, 1975)
An implicature, the supposition of which is necessary for maintaining the assumption that the speaker is cooperative.

1. Had sp. believed John likes red, she should have said so.
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   \( \ldots \) (‘the epistemic step’ - Sauerland, 2004)
3. She believes that he doesn’t like red.

“[the epistemic] step does not follow from Gricean maxims and logic alone.” - Chierchia, et al. (2008)

Wrong, it does!
Outline

1. Diagnosis
2. Solution
3. Conclusion
4. Related concepts and puzzles
1. Diagnosis

(2) a. Which colours (among red, green and blue) does John like?  
b. He likes blue. \[\sim He\ doesn’t\ like\ red\]  
c. He likes blue, or blue and red. \[\sim\sim He\ doesn’t\ like\ red\]
1. Diagnosis

\[(2)\] a. Which colours (among red, green and blue) does John like?
b. He likes blue. \(\sim \) He doesn’t like red
c. He likes blue, or blue and red. \(\triangleright \) He doesn’t like red

Intuition

(2b) and (2c) differ in their \textit{attentive content}.
1. Diagnosis

(2) a. Which colours (among red, green and blue) does John like?
   b. He likes blue.  \(\sim\) He doesn’t like red
   c. He likes blue, or blue and red.  \(\not\sim\) He doesn’t like red

Intuition
(2b) and (2c) differ in their *attentive content*.

\(\triangleright\) (2c) draws attention to the poss. that John likes blue and red.
1. Diagnosis

(2) a. Which colours (among red, green and blue) does John like?
   b. He likes blue. ~ He doesn’t like red
   c. He likes blue, or blue and red. ~\,\,\,✓ He doesn’t like red

Intuition
(2b) and (2c) differ in their attentive content.

» (2c) draws attention to the poss. that John likes blue and red.
» (And so does (2a).)
1. Diagnosis

(2) a. Which colours (among red, green and blue) does John like?
   b. He likes blue.  \(\sim\) He doesn’t like red
   c. He likes blue, or blue and red.  \(\not\sim\) He doesn’t like red

Intuition
(2b) and (2c) differ in their *attentive content*.

- (2c) draws attention to the poss. that John likes blue and red.
- (And so does (2a).)
- (2b) doesn’t; it leaves the possibility *unattended*. 
1. Diagnosis

(2) a. Which colours (among red, green and blue) does John like? 
b. He likes blue.  \( \sim \) He doesn’t like red 
c. He likes blue, or blue and red. \( \sim \) He doesn’t like red

Intuition
(2b) and (2c) differ in their attentive content.

- (2c) draws attention to the poss. that John likes blue and red.
- (And so does (2a).)
- (2b) doesn’t; it leaves the possibility unattended.

Apparently, pragmatic reasoning is sensitive to this.
1. Diagnosis

(2) a. Which colours (among red, green and blue) does John like?
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Intuition
(2b) and (2c) differ in their\textit{attentive content}.

\begin{itemize}
  \item (2c) draws attention to the poss. that John likes blue and red.
  \item (And so does (2a).)
  \item (2b) doesn’t; it leaves the possibility \textit{unattended}.
\end{itemize}

Apparently, pragmatic reasoning is sensitive to this.
1. Diagnosis

(2) a. Which colours (among red, green and blue) does John like?
   b. He likes blue.          \(\sim\) He doesn’t like red
   c. He likes blue, or blue and red.    \(\sim\) He doesn’t like red

Intuition

(2b) and (2c) differ in their **attentive content**.

- (2c) draws attention to the poss. that John likes blue and red.
- (And so does (2a).)
- (2b) doesn’t; it leaves the possibility **unattended**.

Apparently, pragmatic reasoning is sensitive to this.
1. Diagnosis

(2) a. Which colours (among red, green and blue) does John like?
   b. He likes blue. ~ He doesn’t like red
   c. He likes blue, or blue and red. ~

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(2b) and (2c) differ in their attentive content.

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1. Diagnosis

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   b. He likes blue.  ~ He doesn’t like red
   c. He likes blue, or blue and red.  ~ He doesn’t like red

Intuition
(2b) and (2c) differ in their attentive content.
   ▶ (2c) draws attention to the poss. that John likes blue and red.
   ▶ (And so does (2a).)
   ▶ (2b) doesn’t; it leaves the possibility unattended.

Apparently, pragmatic reasoning is sensitive to this.
2. Solution

2.1. Translation into logic
2.2. Semantics
2.3. Pragmatics
2.4. Predictions
2.1. Translation into logic

(3) a. Which colours (among red, green and blue) does John like?
   b. He likes blue. \( \sim \) He doesn’t like red
   c. He likes blue, or blue and red. \( \lor \) He doesn’t like red
(3) a. Which colours (among red and blue) does John like?
   b. He likes blue. \( \sim \) He doesn’t like red
   c. He likes blue, or blue and red. \( \lor \) He doesn’t like red
2.1. Translation into logic

(3) a. There are colours (among red and blue) that John likes.
   b. He likes blue.          ∼ He doesn’t like red
   c. He likes blue, or blue and red.  ⊤ He doesn’t like red
2.1. Translation into logic

(3) a. John likes blue, red, or blue and red.
   b. He likes blue. \( \sim \) He doesn’t like red
   c. He likes blue, or blue and red. \( \nabla \) He doesn’t like red
2.1. Translation into logic

(3) a. John likes blue, red, or blue and red. \[ p \lor q \lor (p \land q) \]
b. He likes blue. \[ p \]
c. He likes blue, or blue and red. \[ p \lor (p \land q) \]
2.2. Semantics (Roelofsen, 2011)

Possibility: a set of worlds $(a, b)$

Proposition: a set of possibilities $(A, B, [\phi])$

Informative content: $\phi \divides \phi$

Entailment: $A$ entails $B$, $A \nrightarrow B$, iff (i) $A \subseteq B$; and (ii) for all $b \in B$, if $b \cap A \neq \emptyset$, $b \cap A \in A$. Now, (3c) $\nrightarrow$ (3a), but (3b) $/\\leftarrow$ (3a).
2.2. Semantics (Roelofsen, 2011)

- **Possibility:** a set of worlds \((a, b)\)
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- **Possibility**: a set of worlds \((a, b)\)
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- **Possibility**: a set of worlds \((a, b)\)
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- **Informative content**: \(|\varphi| := \bigcup[\varphi]\)
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- **Possibility**: a set of worlds \((a, b)\)
- **Proposition**: a set of possibilities \((A, B, [\varphi])\)
- **Informative content**: \(|\varphi| := \bigcup[\varphi]|\)

\[
\begin{align*}
(3a) \quad [p \lor q \lor (p \land q)] & \quad (3b) \quad [p] & \quad (3c) \quad [p \lor (p \land q)]
\end{align*}
\]
2.2. Semantics (Roelofsen, 2011)

- *Possibility*: a set of worlds \((a, b)\)
- *Proposition*: a set of possibilities \((A, B, [\varphi])\)
- *Informative content*: \(|\varphi| := \bigcup [\varphi] \)

\[
(3a) \quad [p \lor q \lor (p \land q)]
\]

\[
(3b) \quad [p]
\]

\[
(3c) \quad [p \lor (p \land q)]
\]
2.2. Semantics (Roelofsen, 2011)

- **Possibility**: a set of worlds \((a, b)\)
- **Proposition**: a set of possibilities \((A, B, [\varphi])\)
- **Informative content**: \(|\varphi| := \bigcup [\varphi]\)

Entailment

\(A \text{ entails } B, A \models B, \text{ iff}\)

(i) \(\bigcup A \subseteq \bigcup B\); and

(ii) for all \(b \in B\), if \(b \cap \bigcup A \neq \emptyset\), \(b \cap \bigcup A \in A\)
2.2. Semantics (Roelofsen, 2011)

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\begin{align*}
(3a) & \quad [p \lor q \lor (p \land q)] \\
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**Entailment**

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\(\rightarrow \text{at least as informative}\)
2.2. Semantics (Roelofsen, 2011)

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- **Proposition**: a set of possibilities \((A, B, [\varphi])\)
- **Informative content**: \(|\varphi| := \bigcup[\varphi]\

\[
\begin{align*}
\text{(3a)} & \ [p \lor q \lor (p \land q)] \\
\text{(3b)} & \ [p] \\
\text{(3c)} & \ [p \lor (p \land q)]
\end{align*}
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**Entailment**

\(A \text{ entails } B, \ A \models B, \) iff

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\[\rightarrow \text{at least as informative} \]

\[\rightarrow \text{at least as attentive}\]
2.2. Semantics (Roelofsen, 2011)

- **Possibility**: a set of worlds \((a, b)\)
- **Proposition**: a set of possibilities \((A, B, [\varphi])\)
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\begin{align*}
(3a) & \quad [p \lor q \lor (p \land q)] \\
(3b) & \quad [p] \\
(3c) & \quad [p \lor (p \land q)]
\end{align*}
\]

**Entailment**

\(A\) entails \(B\), \(A \models B\), iff

(i) \(\bigcup A \subseteq \bigcup B\); and
(ii) for all \(b \in B\), if \(b \cap \bigcup A \neq \emptyset\), \(b \cap \bigcup A \in A\)

Now, \((3c) \models (3a)\), but \((3b) \not\models (3a)\).
2.3. Pragmatics

The relevant maxims

1. Quality:
2. Quantity:
3. Relation:
2.3. Pragmatics

The relevant maxims
For a cooperative speaker with information $s$, responding $R$ to $Q$:

1. Quality:
2. Quantity:
3. Relation:
2.3. Pragmatics

The relevant maxims

For a cooperative speaker with information $s$, responding $R$ to $Q$:

1. **Quality**: $s \subseteq \bigcup R$.
2. **Quantity**:
3. **Relation**:

(4) Did John go to the party?
It was raining.

If it rained, John {did / didn't} go.
2.3. Pragmatics

The relevant maxims
For a cooperative speaker with information $s$, responding $R$ to $Q$:

1. **Quality**: $s \subseteq \bigcup R$.
2. **Quantity**: For all $Q' \subseteq Q$, if $s \subseteq \bigcup Q'$ then $\bigcup R \subseteq \bigcup Q'$.
3. **Relation**: 

$\{r \cap s \mid r \in R\} \cap Q$.
The relevant maxims

For a cooperative speaker with information $s$, responding $R$ to $Q$:

1. Quality: $s \subseteq \bigcup R$.
2. Quantity: For all $Q' \subseteq Q$, if $s \subseteq \bigcup Q'$ then $\bigcup R \subseteq \bigcup Q'$.
3. Relation: $\{ r \cap s \mid r \in R \} \models Q$. 

(4) Did John go to the party?  

It was raining. 

If it rained, John \{did / didn’t\} go.
2.3. Pragmatics

The relevant maxims
For a cooperative speaker with information $s$, responding $R$ to $Q$:

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2. **Quantity**: For all $Q' \subseteq Q$, if $s \subseteq \bigcup Q'$ then $\bigcup R \subseteq \bigcup Q'$.
3. **Relation**: $\{ r \cap s \mid r \in R \} \models Q$.

(4) Did John go to the party?
   It was raining.
The relevant maxims
For a cooperative speaker with information $s$, responding $R$ to $Q$:

1. **Quality**: $s \subseteq \bigcup R$.
2. **Quantity**: For all $Q' \subseteq Q$, if $s \subseteq \bigcup Q'$ then $\bigcup R \subseteq \bigcup Q'$.
3. **Relation**: $\{ r \cap s \mid r \in R \} \equiv Q$.

(4) Did John go to the party?
It was raining.

\[ \begin{array}{cccc}
\{ rp, rp \} & \{ rp, rp \} \\
\{ rp, rp \} & \{ rp, rp \} \\
\end{array} \]
2.3. Pragmatics

The relevant maxims
For a cooperative speaker with information $s$, responding $R$ to $Q$:

1. **Quality**: $s \subseteq \bigcup R$.
2. **Quantity**: For all $Q' \subseteq Q$, if $s \subseteq \bigcup Q'$ then $\bigcup R \subseteq \bigcup Q'$.
3. **Relation**: $\left\{ r \cap s \mid r \in R \right\} \models Q$.

(4) Did John go to the party?
It was raining.
2.3. Pragmatics

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For a cooperative speaker with information $s$, responding $R$ to $Q$:

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3. **Relation**: $\{ r \cap s \mid r \in R \} \models Q$.

(4) Did John go to the party?
It was raining.

\[
\begin{array}{c}
\begin{array}{c}
\text{rp} \\
\text{rp} \\
\text{rp}
\end{array}
\end{array}
= \begin{array}{c}
\begin{array}{c}
\text{rp} \\
\text{rp}
\end{array}
\end{array}
\models \begin{array}{c}
\begin{array}{c}
\text{rp} \\
\text{rp}
\end{array}
\end{array}
\]
2.3. Pragmatics

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For a cooperative speaker with information $s$, responding $R$ to $Q$:

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2. **Quantity**: For all $Q' \subseteq Q$, if $s \subseteq \bigcup Q'$ then $\bigcup R \subseteq \bigcup Q'$.
3. **Relation**: $\{ r \cap s \mid r \in R \} \models Q$.

(4) Did John go to the party?
   It was raining.
2.3. Pragmatics

The relevant maxims
For a cooperative speaker with information $s$, responding $R$ to $Q$:

1. **Quality**: $s \subseteq \bigcup R$.
2. **Quantity**: For all $Q' \subseteq Q$, if $s \subseteq \bigcup Q'$ then $\bigcup R \subseteq \bigcup Q'$.
3. **Relation**: $\{r \cap s \mid r \in R\} \models Q$.

(4) Did John go to the party?
It was raining.
The relevant maxims

For a cooperative speaker with information $s$, responding $R$ to $Q$:

1. **Quality**: $s \subseteq \bigcup R$.
2. **Quantity**: For all $Q' \subseteq Q$, if $s \subseteq \bigcup Q'$ then $\bigcup R \subseteq \bigcup Q'$.
3. **Relation**: $\{r \cap s \mid r \in R\} \models Q$.

(4) Did John go to the party?
   It was raining. $\Rightarrow$ If it rained, John \{did / didn’t\} go.
2.3. Pragmatics

The relevant maxims
For a cooperative speaker with information $s$, responding $R$ to $Q$:

1. **Quality**: $s \subseteq \bigcup R$.
2. **Quantity**: For all $Q' \subseteq Q$, if $s \subseteq \bigcup Q'$ then $\bigcup R \subseteq \bigcup Q'$.
3. **Relation**: $\{ r \cap s \mid r \in R \} \models Q$. 

(4) Did John go to the party?
It was raining.
If it rained, John \{did / didn't\} go.
2.3. Pragmatics
(cf. Groenendijk and Stokhof, 1984; Roberts, 1996; Spector, 2007)

The relevant maxims
For a cooperative speaker with information $s$, responding $R$ to $Q$:

1. **Quality**: $s \subseteq \bigcup R$.
2. **Quantity**: For all $Q' \subseteq Q$, if $s \subseteq \bigcup Q'$ then $\bigcup R \subseteq \bigcup Q'$.
3. **Relation**: $\{ r \cap s \mid r \in R \} \vdash Q$.

(4) Did John go to the party?
It was raining.
\[ \text{If it rained, John } \{ \text{did }/\text{ didn't} \} \text{ go.} \]
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

   b. He likes blue. \((p)\)

   c. He likes blue, or blue and red. \((p \lor (p \land q))\)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)

c. He likes blue, or blue and red. \((p \lor (p \land q))\)

1. \(s \subseteq |p \lor (p \land q)|\) (Quality)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

   b. He likes blue. \((p)\)

c. He likes blue, or blue and red. \((p \lor (p \land q))\)

   1. \(s \subseteq |p \lor (p \land q)| = |p|\) (Quality)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)

c. He likes blue, or blue and red. \((p \lor (p \land q))\)

1. \(s \subseteq |p \lor (p \land q)| = |p|\) (Quality)
2. \(s \notin |q|\) (Quantity)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)

c. He likes blue, or blue and red. \((p \lor (p \land q))\)

1. \(s \subseteq |p \lor (p \land q)| = |p|\) (Quality)
2. \(s \not\subseteq |q|\) (Quantity)

\(p \lor (p \land q) \models p \lor q \lor (p \land q)\)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)

c. He likes blue, or blue and red. \((p \lor (p \land q))\)

1. \(s \subseteq |p \lor (p \land q)| = |p|\) (Quality)
2. \(s \not\subseteq |q|\) (Quantity)
3. \(- \ p \lor (p \land q) \models p \lor q \lor (p \land q)\) (Relation)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)
1. \(s \subseteq |p|\)  

(c. He likes blue, or blue and red. \((p \lor (p \land q))\)
1. \(s \subseteq |p \lor (p \land q)| = |p|\)  
2. \(s \not\subseteq |q|\)  
3. \(p \lor (p \land q) \models p \lor q \lor (p \land q)\)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)

1. \(s \subseteq |p|\)  
2. \(s \notin |q|\)


c. He likes blue, or blue and red. \((p \lor (p \land q))\)

1. \(s \subseteq |p \lor (p \land q)| = |p|\)  
2. \(s \notin |q|\)  
3. \(p \lor (p \land q) \models p \lor q \lor (p \land q)\)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)
   1. \(s \subseteq |p|\)
   2. \(s \notin |q|\)

!!!(Quality)
!!!(Quantity)

---

(3) c. He likes blue, or blue and red. \((p \lor (p \land q))\)
   1. \(s \subseteq |p \lor (p \land q)| = |p|\)
   2. \(s \notin |q|\)
   3. \(p \lor (p \land q) \supseteq p \lor q \lor (p \land q)\)

   (Quality)
   (Quantity)
   (Relation)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)
   1. \(s \subseteq |p|\)
   2. \(s \not\subseteq |q|\)

   \[\begin{array}{ll}
   & p \lor q \lor (p \land q) \\
   \end{array}\]
   (Quality)
   (Quantity)

   \[\begin{array}{ll}
   & p \\
   \end{array}\]

   \[\begin{array}{ll}
   & q \\
   \end{array}\]

   \[\begin{array}{ll}
   & q \\
   \end{array}\]
   \[\begin{array}{ll}
   & q \\
   \end{array}\]

   \[\begin{array}{ll}
   & q \\
   \end{array}\]

   \[\begin{array}{ll}
   & q \\
   \end{array}\]

   \[\begin{array}{ll}
   & q \\
   \end{array}\]

   \[\begin{array}{ll}
   & q \\
   \end{array}\]

   \[\begin{array}{ll}
   & q \\
   \end{array}\]

   \[\begin{array}{ll}
   & q \\
   \end{array}\]

   \[\begin{array}{ll}
   & q \\
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2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)
1. \(s \subseteq |p|\)
2. \(s \not\subseteq |q|\)

\(p \not\subseteq p \lor q \lor (p \land q)\)

(Quality)

(Quantity)

c. He likes blue, or blue and red. \((p \lor (p \land q))\)
1. \(s \subseteq |p \lor (p \land q)| = |p|\)
2. \(s \not\subseteq |q|\)
3. \(p \lor (p \land q) \supseteq p \lor q \lor (p \land q)\)

(Quality)

(Quantity)

(Relation)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)

1. \(s \subseteq |p|\)
2. \(s \not\subseteq |q|\)

\[p \not\vDash p \lor q \lor (p \land q)\]

(Quality)

(Quantity)

c. He likes blue, or blue and red. \((p \lor (p \land q))\)

1. \(s \subseteq |p \lor (p \land q)| = |p|\)
2. \(s \not\subseteq |q|\)
3. - \(p \lor (p \land q) \not\vDash p \lor q \lor (p \land q)\)

(Quality)

(Quantity)

(Relation)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)
1. \(s \subseteq |p|\)
2. \(s \notin |q|\)

\[ p \not\models p \lor q \lor (p \land q) \]

(Quality)

(Quantity)

\(\lor\)

\(\land\)

\(\Rightarrow\)

c. He likes blue, or blue and red. \((p \lor (p \land q))\)
1. \(s \subseteq |p \lor (p \land q)| = |p|\)
2. \(s \notin |q|\)
3. \(p \lor (p \land q) \models p \lor q \lor (p \land q)\)

(Quality)

(Quantity)

(Relation)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)
   1. \(s \subseteq |p|\)
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\[ p \not\subseteq p \lor q \lor (p \land q) \]

(Quality)

(Quality)

\(\vdash\)

\(\equiv\)

\(p \lor (p \land q) \equiv p \lor q \lor (p \land q)\)

(Quality)

(Quality)

(Relation)

c. He likes blue, or blue and red. \((p \lor (p \land q))\)
   1. \(s \subseteq |p \lor (p \land q)| = |p|\)
   2. \(s \not\subseteq |q|\)
   3. \(-\)

\(p \lor (p \land q) \models p \lor q \lor (p \land q)\)

(Quality)

(Quality)

(Relation)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)
   1. \(s \subseteq |p|\)
   2. \(s \not\subseteq |q|\)
   3. \(s \subseteq |p| \cup |q|\) or \(s \subseteq |p| \cup |q|\)

   \(\text{!!! p } \not\subseteq p \lor q \lor (p \land q)\)
   (Quality)

   \(\text{!!! q } \not\subseteq p \lor q \lor (p \land q)\)
   (Quantity)

   \(\text{!!! (Relation)}\)

   \(\text{!!! (Quality)}\)

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1. \(s \subseteq |p|\)
2. \(s \notin |q|\)
3. \(s \subseteq |p| \cup |q|\) or \(s \subseteq |p| \cup |q|\)


c. He likes blue, or blue and red. \((p \lor (p \land q))\)

1. \(s \subseteq |p \lor (p \land q)| = |p|\)
2. \(s \notin |q|\)
3. \(-p \lor (p \land q) \models p \lor q \lor (p \land q)\) (Relation)
2.4. Predictions

(3) a. John likes blue, red, or blue and red. \((p \lor q \lor (p \land q))\)

b. He likes blue. \((p)\)
   1. \(s \subseteq |p|\)
   2. \(s \not\subseteq |q|\)
   3. \(s \subseteq |p| \cup |q|\) or \(s \subseteq |p| \cup |q|\)
   4. \(s \subseteq |q|\)

     \[p \not\subseteq p \lor q \lor (p \land q)\]

     \((\text{Quality})\)

     \[p \not\subseteq q\]

     \((\text{Quantity})\)

     \[s \subseteq |p| \cup |q| \lor s \subseteq |p| \cup |q|\]

     \((\text{Relation})\)

   

c. He likes blue, or blue and red. \((p \lor (p \land q))\)
   1. \(s \subseteq |p \lor (p \land q)| = |p|\)
   2. \(s \not\subseteq |q|\)
   3. \(p \lor (p \land q) \supseteq p \lor q \lor (p \land q)\)

     \((\text{Quality})\)

     \(p \lor (p \land q) \supseteq q\)

     \((\text{Quantity})\)

     \[p \lor (p \land q) \supseteq p \lor q \lor (p \land q)\]

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1. \(s \subseteq |p|\)
2. \(s \not\subseteq |q|\)  
3. \(s \subseteq |p| \cup |q|\) or \(s \subseteq |p| \cup |q|\)

4. \(s \subseteq |q|\) exhaustivity!

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(3) c. He likes blue, or blue and red. \((p \lor (p \land q))\)

1. \(s \subseteq |p \lor (p \land q)| = |p|\) \(\) (Quality)
2. \(s \not\subseteq |q|\)  
3. \(p \lor (p \land q) \models p \lor q \lor (p \land q)\) \(\) (Relation)
3. Conclusion

Main finding:
3. Conclusion

Main finding:

- If we feed the maxims *attentive content*
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3. Conclusion

Main finding:
- If we feed the maxims *attentive content*
- - which we must anyway, to distinguish between (3b,3c) -
- then the epistemic step follows from the cooperative principle.

Take-home messages:
- Pragmatic reasoning is sensitive to attentive content.
- *Exhaustivity implicatures are conversational implicatures.*
4. Related concepts and puzzles

4.1. The opinionatedness assumption
4.2. ‘Alternatives’
4.3. ‘Embedded’ implicatures
4.4. Other suitable semantics
4.5. Roberts’s (1996) ‘relevance’
4.6. One-sided/two-sided numerals
4.1. The opinionatedness assumption

Most existing work (Sauerland, 2004):

1. The speaker doesn't believe $q$.
2. She believes either $q$ or $\neg q$.

Counterexample:
(5) I'm asking the wrong person, but which colours does J. like?
He likes blue.
He doesn't like red.

Instead, in my approach:
Opinionatedness follows from Quality + Relation implicatures.
4.1. The opinionatedness assumption

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4.1. The opinionatedness assumption

Most existing work (Sauerland, 2004):

1. The speaker doesn’t believe $q$  
   (Quantity)
2. She believes either $q$ or $\neg q$  
   (Context)

Counterexample: 
(5) I’m asking the wrong person, but which colours does J. like?  
He likes blue.  
$\neg$He doesn’t like red.
4.1. The opinionatedness assumption

Most existing work (Sauerland, 2004):

1. The speaker doesn’t believe $q$  
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2. She believes either $q$ or $\neg q$  
   (Context)

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He likes blue. \( \sim \) He doesn’t like red.
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4.2. ‘Alternatives’

Existing approaches (since Gazdar, 1979?):

- ‘Why did the speaker not say “$p \land q$”?’
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Existing approaches (since Gazdar, 1979?):
- ‘Why did the speaker not say “p ∧ q”?’
- Mere ignorance is sufficient reason.
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My approach:
  - ‘Why did the speaker not say “p \lor (p \land q)”?’
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Existing approaches (since Gazdar, 1979?):

- ‘Why did the speaker not say “p ∧ q”? ’
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My approach:

- ‘Why did the speaker not say “p ∨ (p ∧ q)”?’
- *Ignorance is no excuse.*
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- Hence something stronger is implied: exhaustivity.
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My approach:
- ‘Why did the speaker not say “p \lor (p \land q)”?’
- *Ignorance is no excuse.*
- Hence something stronger is implied: exhaustivity.

More take-home messages
- The ‘alternatives’ are fully determined by the maxims.
- Speakers need not reason in terms of alternatives.
4.3. ‘Embedded’ implicatures
Chierchia, *et al.* (2008), and much subsequent discussion

(6) Which books (among O. and K.L.) did every student read?
Every student read O. or K.L. $\sim$ No student read both.
4.3. ‘Embedded’ implicatures
Chierchia, et al. (2008), and much subsequent discussion

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The problem
The problem has never been the Gricean approach as such, but rather to find the right ‘alternatives’.
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In the present theory:

- The maxims are sensitive to attentive content
- Attentive content mirrors sub-sentential structure.
- (Hence so do the ‘alternatives’.)

The ‘embedded’ implicature of (6) is in fact predicted.
4.4. Other suitable semantics

Attentive semantics is not the only suitable semantics:
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- *Unrestricted Inquisitive Sem.* (Ciardelli, 2009; Westera, 2012)
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Minimally, the semantics must lack the *absorption laws*:

- Absorption: $p \lor (p \land q) \equiv p \equiv p \land (p \lor q)$
4.4. Other suitable semantics

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4.5. Roberts’s (1996) ‘relevance’

- ‘My’ Maxim of Relation: \( R_s \vDash Q \)
- Roberts’s relevance: \( R_{CG} \vDash Q \) \((CG = \text{Common Ground})\)
4.5. Roberts’s (1996) ‘relevance’

- ‘My’ Maxim of Relation: $R_s \models Q$
- Roberts’s relevance: $R_{CG} \models Q$  \hspace{1cm} (CG = Common Ground)

Roberts's requirement is too strong:
4.5. Roberts’s (1996) ‘relevance’

- ‘My’ Maxim of Relation: $R_s \models Q$
- Roberts’s relevance: $R_{CG} \models Q$ \hspace{1cm} ($CG = \text{Common Ground}$)

Roberts’s requirement is too strong:
- The participants need not already know how $R$ is relevant.
4.5. Roberts’s (1996) ‘relevance’

- ‘My’ Maxim of Relation: $R_s \models Q$
- Roberts’s *relevance*: $R_{CG} \models Q$  \hspace{1cm} (CG = Common Ground)

Roberts’s requirement is too strong:

- The participants need not *already know* how $R$ is relevant.
- They need only be able to *figure it out*. 

\begin{align*}
&1. s \subseteq p \quad (Quality) \\
&2. s \subseteq q \quad (Quantity) \\
&3. s \subseteq p \cup q \quad (Relation) \\
&4. s \subseteq q
\end{align*}
4.5. Roberts’s (1996) ‘relevance’

- ‘My’ Maxim of Relation: $R_s \vDash Q$
- Roberts’s *relevance*: $R_{CG} \vDash Q$ \quad (CG = Common Ground)

Roberts's requirement is too strong:
- The participants need not *already know* how $R$ is relevant.
- They need only be able to *figure it out*.

E.g., in case of exhaustivity:

1. $s \subseteq |p|$ \quad (Quality)
2. $s \notin |q|$ \quad (Quantity)
3. $s \subseteq \overline{|p| \cup |q|}$ or $s \subseteq |p| \cup |q|$ \quad (Relation)
4. $s \subseteq \overline{|q|}$
4.6. One-sided/two-sided numerals

(7) a. There are three apples.
    b. There are at least three apples.
    c. There are exactly three apples.
4.6. One-sided/two-sided numerals

(7) a. There are three apples.
   b. There are at least three apples.
   c. There are exactly three apples.

Does (7a) mean (7b) (‘one-sided’) or (7c) (‘two-sided’)?
4.6. One-sided/two-sided numerals

(7) a. There are three apples.
   b. There are at least three apples.
   c. There are exactly three apples.

Does (7a) mean (7b) ('one-sided') or (7c) ('two-sided')? Neither!
4.6. One-sided/two-sided numerals

(7) a. There are three apples. \( \exists x. A x \land |x| = 3 \)
    b. There are at least three apples. \( \exists x. A x \land |x| \geq 3 \)
    c. There are exactly three apples. \( \exists! x. A x \land |x| = 3 \)

Does (7a) mean (7b) (‘one-sided’) or (7c) (‘two-sided’)? \textcolor{red}{Neither!}
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Does (7a) mean (7b) (‘one-sided’) or (7c) (‘two-sided’)? **Neither!**

(7a) \[ \begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\end{array} \]

(7b) \[ \begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\end{array} \]

(7c) \[ \begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\end{array} \]
4.6. One-sided/two-sided numerals
(cf. Coppock and Brochhagen, 2013)

\[(7)\]

a. There are three apples. \[\exists x. A x \land |x| = 3\]
b. There are at least three apples. \[\exists x. A x \land |x| \geq 3\]
c. There are exactly three apples. \[\exists! x. A x \land |x| = 3\]

Does (7a) mean (7b) (‘one-sided’) or (7c) (‘two-sided’)? \textcolor{red}{Neither!}
Fin.

Contact
Matthijs Westera - m.westera@uva.nl

Article

  *ESSLLI StuS proceedings* (staff.science.uva.nl/~westera/)

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Appendix A. Semantics (Roelofsen, 2011)

Ingredients

- Possibility: a set of worlds \((a, b)\)
- Proposition: a set of possibilities \((A, B, [\varphi])\)
- Informative content: \(|\varphi| := \cup[\varphi]\)
- A restricted to \(b\), \(A_b := \{a \cap b \mid a \in A, a \cap b \neq \emptyset\}\)

Semantics of relevant fragment

1. \([p] = \\{\{w \in \text{Worlds} \mid w(p) = \text{true}\}\}\)
2. \([\varphi \lor \psi] = ([\varphi] \cup [\psi])\mid_{\varphi\cup\psi} = [\varphi] \cup [\psi]\)
3. \([\varphi \land \psi] = ([\varphi] \cup [\psi])\mid_{\varphi\cap\psi}\

Entailment

\(A \text{ entails } B, A \models B, \text{ iff (i) } \bigcup A \subseteq \bigcup B \text{ and (ii) } B \cup A \subseteq A.\)
Appendix B. The final rise
To be presented at ESSLLI.

(7) Which colours (among red, green and blue) does John like?
He likes blue↑.
Appendix B. The final rise
To be presented at ESSLLI.

(7) Which colours (among red, green and blue) does John like? He likes blue ↗.

Conveys uncertainty regarding:
- whether he really likes blue
Appendix B. The final rise
To be presented at ESSLLI.

(7) Which colours (among red, green and blue) does John like?
He likes blue↗.

Conveys uncertainty regarding:

- whether he really likes blue
- whether this is sufficient info
Appendix B. The final rise
To be presented at ESSLLI.

(7) Which colours (among red, green and blue) does John like? He likes blue.

Conveys uncertainty regarding:
- whether he really likes blue
- whether this is sufficient info
- whether ‘blue’ is pronounced correctly
Appendix B. The final rise

To be presented at ESSLLI.

(7) Which colours (among red, green and blue) does John like?
   He likes blue ↗.

Conveys uncertainty regarding:

- whether he really likes blue
- whether this is sufficient info
- whether ‘blue’ is pronounced correctly
- whether he likes red
Appendix B. The final rise
To be presented at ESSLLI.

(7) Which colours (among red, green and blue) does John like? He likes blue ↗.

Conveys uncertainty regarding:

- whether he really likes blue
- whether this is sufficient info
- whether ‘blue’ is pronounced correctly
- whether he likes red

Proposal
The final rise conveys uncertain cooperativity.
Appendix B. The final rise
To be presented at ESSLLI.

(7) Which colours (among red, green and blue) does John like? He likes blue ↗.

Conveys uncertainty regarding:
- whether he really likes blue (Quality)
- whether this is sufficient info
- whether ‘blue’ is pronounced correctly
- whether he likes red

Proposal
The final rise conveys uncertain cooperativity.
(7) Which colours (among red, green and blue) does John like? He likes blue ↗.

Conveys uncertainty regarding:

- whether he really likes blue (Quality)
- whether this is sufficient info (Quantity)
- whether ‘blue’ is pronounced correctly
- whether he likes red

Proposal

The final rise conveys uncertain cooperativity.
Appendix B. The final rise
To be presented at ESSLLI.

(7) Which colours (among red, green and blue) does John like?
   He likes blue↗.

Conveys uncertainty regarding:
   ▶ whether he really likes blue                      (Quality)
   ▶ whether this is sufficient info                 (Quantity)
   ▶ whether ‘blue’ is pronounced correctly          (Manner)
   ▶ whether he likes red

Proposal
The final rise conveys uncertain cooperativity.
Appendix B. The final rise
To be presented at ESSLLI.

(7) Which colours (among red, green and blue) does John like?
He likes blue ↗.

Conveys uncertainty regarding:
- whether he really likes blue (Quality)
- whether this is sufficient info (Quantity)
- whether ‘blue’ is pronounced correctly (Manner)
- whether he likes red (Relation)

Proposal
The final rise conveys uncertain cooperativity.
Appendix C. References