

Unrestricted inquisitive semantics and discourse coherence*

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Abstract

Within the framework of inquisitive semantics, we investigate the semantic prerequisites of an account of discourse coherence. In inquisitive semantics two views on meaning exist. *Basic inquisitive semantics*, InqB , follows from the view that to utter a sentence is to provide and request information (Roelofsen, 2011). *Unrestricted inquisitive semantics*, InqU , follows from the view that to utter a sentence is to propose to update the common ground in any of several ways (Ciardelli, Groenendijk, & Roelofsen, 2009). We illustrate with a simple example that InqU , but not InqB , can be a semantic foundation for an account of discourse coherence.

However, the clauses of InqU have not been motivated conceptually with as much rigour as those of InqB , and they are technically not as well understood. In this paper we precise its conceptual motivation and, based on this conception of meaning, define a semantics driven by general algebraic concerns. We show that the algebraic backbone of InqU is a commutative, idempotent semiring, which will facilitate an integration of inquisitive semantics with other formalisms. The algebraic structure gives rise to a compliance order on meanings that we put forward as a core notion for an account of discourse coherence.

1 Introduction

The adverb ‘actually’ can indicate some kind of incoherence or unexpectedness in a discourse. Consider observation 1. Bob’s response in example 1 can be marked with ‘actually’, indicating that his response was not expected after Ann’s initiative (at least in some contexts, with some intonations, etcetera). This contrasts with Ann’s initiative in example 2, to which the same response by Bob clearly can *not* be marked with ‘actually’, indicating that Bob’s response was expected after Ann’s initiative.

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Observation 1 (The Challenge)

1. Ann: *John will go to the party, or Mary.*
Bob: *Actually, both will go.*
2. Ann: *John will go to the party, or Mary, or both.*
Bob: *# Actually, both will go.*

One can try to explain the contrast between examples 1 and 2 in various ways (e.g., perhaps by means of pragmatic strengthening or by means of semantic focus), define a general notion of discourse coherence and give a precise semantics of the word ‘actually’. But this is not our aim in this paper.

Rather, our aim is to lay the *semantic foundation* for such an account. Any semantic or pragmatic account of the contrast in observation 1 is hopeless if Ann’s initiatives in each are not already semantically distinct to begin with (and we will assume that a semantic/pragmatic explanation is preferable over a syntactic one). I.e., the semantics at the basis should at least assign different meanings to the formulae $p \vee q$ and $p \vee q \vee (p \wedge q)$. We will refer to this challenge as ‘the Challenge’.

In light of the Challenge, we consider two implementations of inquisitive semantics. First, in *basic inquisitive semantics* (InqB) (Roelofsen, 2011, see also Ciardelli & Roelofsen, 2011; Groenendijk & Roelofsen, 2009), the meaning of a sentence embodies the information it provides and the issues it raises. We call this view on meaning ‘the Basic View’. Starting with the Basic View, the definition of InqB follows entirely from general algebraic concerns (Roelofsen, 2011). Second, *unrestricted inquisitive semantics* (InqU) (Ciardelli, 2010; Ciardelli et al., 2009) is based on a view of meanings as *proposals* to update the common ground in any of several ways, or as *drawing attention* to those possibilities. We call this view on meaning ‘the Proposal View’. The clauses of InqU have not been motivated conceptually with as much rigour as those of InqB , and they are technically not as well understood.

We will see that InqU , but not InqB , resolves the Challenge. Hence, the Basic View on meaning is unfit for a theory of discourse coherence, while the Proposal View is more adequate. This is our motivation to investigate the conceptual and technical underpinnings of InqU . We start by giving a concise overview of InqB in section 2. In section 3 we define InqU from scratch, starting from the Proposal View on meaning and driven by general algebraic concerns. The resulting algebraic structure, a commutative, idempotent semiring, gives rise to a compliance order on meanings that may be a suitable core notion for an account of discourse coherence.

The resulting definition of InqU will largely coincide with the definition in (Ciardelli, 2010; Ciardelli et al., 2009), the only substantial difference being the clause for implication. Therefore, we take our main contribution to be not the semantics itself, but a deeper conceptual and algebraic understanding of it and its role in an account of discourse coherence.

2 InqB: Basic inquisitive semantics

2.1 Preliminaries and the Basic View of meaning

For InqB, we follow the exposition in (Roelofsen, 2011).¹ We consider only the language of propositional logic:

Definition 1 (Syntax) For φ ranging over formulae, π over proposition letters:
 $\varphi := \pi \mid \perp \mid (\varphi \vee \varphi) \mid (\varphi \wedge \varphi) \mid (\varphi \rightarrow \psi)$, with $\neg\varphi := \varphi \rightarrow \perp$.

The semantics for this language is defined relative to a suitable model:

Definition 2 (Model) A model M is a tuple $\langle \mathbf{W}, I \rangle$, where \mathbf{W} is a set of worlds and I is an interpretation function that, relative to a possible world, maps each proposition letter to a truth value.

Based on a model, an epistemic state is defined as any subset of the set of possible worlds of the model:

Definition 3 (Epistemic state) An epistemic state based on the model $\langle \mathbf{W}, I \rangle$ is a set $s \subseteq \mathbf{W}$.

The meaning of a formula, i.e., its *proposition*, is a set of epistemic states (this will shortly be refined by adding some restrictions):

Definition 4 (Proposition [to be refined]) A proposition based on the model $\langle \mathbf{W}, I \rangle$ is a set of states based on $\langle \mathbf{W}, I \rangle$.

Propositions are conceptualised in a particular way. In InqB, the meaning of a formula embodies both its informative content and its inquisitive content. We call this the *Basic View*:

Definition 5 (The Basic View) A speaker, in expressing a sentence denoting proposition A , provides the information that the actual world is in one of the states, i.e., in $\cup A$ (informative content), and requests sufficient information to locate the actual world in one of the states in A (inquisitive content).

Because the term ‘sufficient information’ is monotone with information growth, any state in a proposition that is properly contained in another state in the proposition, is irrelevant for the informative and inquisitive content of the proposition. This means that we should restrict propositions either to sets of only *maximal* states, or to *downward-closed* sets of states, i.e., sets of states containing all substates. We impose the second restriction, downward closure (the two restrictions in fact yield equivalent results; for details see (Roelofsen, 2011)).

¹See www.illc.uva.nl/inquisitive-semantics for an overview of inquisitive semantics, published work and manuscripts.

Definition 6 (Proposition) A proposition based on the model $\langle \mathbf{W}, I \rangle$ is a downward-closed, non-empty set of states based on $\langle \mathbf{W}, I \rangle$, i.e., for all states $s \in A$, all substates $t \subseteq s$ are also in A . Let $\mathbf{\Pi}$ be the set of all propositions (based on a given model).

We wish to stress that the downward closure restriction follows directly from the Basic View; just like the rest of the semantics below. (The non-emptiness requirement in this definition is a technicality that makes the definitions run smoothly.) We write $\llbracket \varphi \rrbracket$ for the proposition of a formula φ .

To illustrate, figure 1 shows the proposition $\llbracket p \vee q \rrbracket$, for a model that consists of four worlds (small circles), that differ with respect to two proposition letters p and q ('10' indicates that p is true and q is false, '11' that both are true, etc.). All rounded rectangles/squares represent states. The proposition $\llbracket p \vee q \rrbracket$ embodies the information that p or q holds, and the request for sufficient information to decide whether p or q holds.

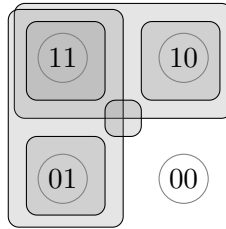


Figure 1: A visualisation of the proposition $\llbracket p \vee q \rrbracket$.

2.2 Algebraic characterisation and semantics

A proposition A should *entail* a proposition B iff (i) A provides at least as much information as B and (ii) A requests at least as much information as B . This is the case precisely iff A is contained in B (for more details and proof, see (Roelofsen, 2011)):

Definition 7 (Entailment) For any $A, B \in \mathbf{\Pi}$, A entails B , $A \vDash B$, iff $A \subseteq B$.

Entailment is a partial order, i.e., it is reflexive, transitive and antisymmetric (it would not have been antisymmetric without the downward closure restriction on propositions). The set of all propositions combined with the entailment order forms a *lattice*, and in particular a *Heyting algebra* (the algebra underlying intuitionistic logic).

Fact 1 (Algebraic characterisation of $\mathbf{\Pi}$) $\langle \mathbf{\Pi}, \vDash \rangle$ is a Heyting algebra.

A Heyting algebra comes equipped with three basic operations on propositions, among which are *addition* (\oplus) and *multiplication* (\otimes). In the case of a Heyting Algebra, addition (\oplus) gives the *join*, i.e., the least (i.e., strongest) upper bound, with respect to the entailment order. Multiplication (\otimes) gives the *meet*, i.e., the greatest lower bound. In

the Heyting algebra of propositions, addition corresponds to set union and multiplication corresponds to set intersection.

Unlike a Boolean algebra, there is no complementation operation in a Heyting algebra. There is, however, a *relative pseudo-complementation* operation (\otimes). The relative pseudo-complement $A \otimes B$ is the entailment-greatest (i.e., weakest) proposition C such that $A \otimes C \vDash B$, and its definition is slightly more involved (see below). Summing up, the operations of the Heyting algebra $\langle \mathbf{\Pi}, \vDash \rangle$ are defined as follows:

Definition 8 (Basic operations on $\mathbf{\Pi}$)

1. $A \oplus B = A \cup B$;
2. $A \otimes B = A \cap B$;
3. $A \oslash B = \{s : \forall s' \subseteq s, \text{ if } s' \in A, \text{ then } s' \in B\}$

Associating the basic operations meet, join and relative pseudo-complement with the logical connectives conjunction, disjunction and implication, gives rise to *basic inquisitive semantics* (\mathbf{InqB}):

Definition 9 (\mathbf{InqB})

For π a proposition letter, φ and ψ formulae:

1. $\llbracket \pi \rrbracket = \wp\{w : w(\pi) = 1\}$;
2. $\llbracket \perp \rrbracket = \{\emptyset\}$;
3. $\llbracket \varphi \vee \psi \rrbracket = \llbracket \varphi \rrbracket \oplus \llbracket \psi \rrbracket$;
4. $\llbracket \varphi \wedge \psi \rrbracket = \llbracket \varphi \rrbracket \otimes \llbracket \psi \rrbracket$;
5. $\llbracket \varphi \rightarrow \psi \rrbracket = \llbracket \varphi \rrbracket \oslash \llbracket \psi \rrbracket$.

2.3 The Challenge

In \mathbf{InqB} , the formulae of the Challenge, $p \vee q$ and $p \vee q \vee (p \wedge q)$, denote the exact same proposition (depicted in figure 2):

Fact 2 $\llbracket p \vee q \vee (p \wedge q) \rrbracket = \llbracket p \vee q \rrbracket$.

Since they are semantically equivalent in \mathbf{InqB} , the contrast in observation 1 cannot be explained in any (semantic/pragmatic) way.

Recall that the semantics \mathbf{InqB} follows from a particular view on meaning: the Basic View. Hence, the Basic View, though interesting in its own right, is unfit for a theory of discourse coherence. More technically (but these technicalities, too, follow from the Basic View), the blame can be placed on two (related) properties of \mathbf{InqB} . First, propositions are downward-closed, hence states properly contained in another state cannot make a difference. Second, in a Heyting algebra (or any lattice) the laws of absorption hold, i.e., for all propositions A, B , $A \oplus (A \otimes B) = A$ and $A \otimes (A \oplus B) = A$. From the absorption laws it follows directly that the two formulae of the Challenge are semantically equivalent.

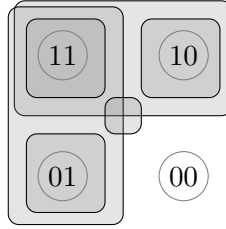


Figure 2: The proposition $\llbracket p \vee q \rrbracket$, identical with $\llbracket p \vee q \vee (p \wedge q) \rrbracket$.

3 InqU: Unrestricted inquisitive semantics

Unrestricted inquisitive semantics (InqU), as defined in (Ciardelli, 2010; Ciardelli et al., 2009), is based on a view of meanings as *proposals* to update the common ground in one of several ways, or, in the same paper and in the same breath, as proposals to take certain possibilities into consideration, or to draw *attention* to those possibilities. The first of these views, of meanings as proposals, actually occurs in early work on InqB, too, although there it did not find its way into the semantics, but only into the pragmatics (e.g., Groenendijk & Roelofsen, 2009). Because it is such a basic conception in inquisitive semantics, because our intuitions regarding ‘proposing to update’ are stronger than for ‘taking into consideration’ and ‘drawing attention to’, and also because we like to flirt with dynamic semantics, we choose to pursue this view - but we suspect that nothing hinges on this.

As we will see, InqU can deal with the Challenge. However, the Proposal View on meaning has not been pursued with great rigour, and the clauses of InqU are not as well-understood as those of InqB. To increase our understanding of the semantics, in this section we define a version of InqU from scratch, starting from the Proposal View and driven by general algebraic considerations.

3.1 Meanings as proposals

Syntax, models and epistemic states are defined exactly as for InqB. Unlike in InqB, we think of meanings as proposals. One does not propose a piece of information; rather, one proposes *doing something* with that information, such as updating the common ground with it. Hence, we define meanings, *proposals*, as sets of functions on epistemic states:

Definition 10 (Proposal [to be refined]) *A proposal based on the model $\langle \mathbf{W}, I \rangle$ is a set of functions on epistemic states based on $\langle \mathbf{W}, I \rangle$, i.e., functions $f : \wp \mathbf{W} \rightarrow \wp \mathbf{W}$.*

Furthermore, we restrict ourselves to functions that are *eliminative* and *distributive*. This allows us to simplify the definition of the resulting semantics, and will make it look and feel like InqB, as well as InqU in (Ciardelli et al., 2009), despite the conceptual shift, as we will see shortly. A function on states is eliminative iff it only *eliminates* worlds, i.e., it does not change the worlds or create new worlds. This means that we consider only functions that model information *growth*, not *loss*; i.e., all functions are actual *update* functions.

Definition 11 (Eliminativity) $f : \wp\mathbf{W} \rightarrow \wp\mathbf{W}$ is eliminative iff $\forall s \subseteq \mathbf{W}, f(s) \subseteq s$.

A function is distributive if we could, instead of applying the function to a state s , apply the function to all singleton substates of s , take the union of their outputs, and obtain the same result. In other words, this means that updates operate locally on worlds, not necessarily globally on states.

Definition 12 (Finite distributivity)

$f : \wp\mathbf{W} \rightarrow \wp\mathbf{W}$ is finitely distributive iff $\forall s, s' \subseteq \mathbf{W}, f(\emptyset) = \emptyset$ and $f(s \cup s') = f(s) \cup f(s')$.

Any eliminative, distributive function can be fully characterised by its effect on the uninformed state \mathbf{W} (Bentham, 1989):

Fact 3 (Update decomposition) For all $f : \wp\mathbf{W} \rightarrow \wp\mathbf{W}$, if f is eliminative and finitely distributive, then for all $s \subseteq \mathbf{W}$, $f(s) = f(\mathbf{W}) \cap s$.

This means that every such update function f corresponds to a unique static object $f(\mathbf{W})$. We will call such objects ‘updates-as-states’, or just ‘updates’ when no confusion can arise. (We do not call them ‘states’, because even though that is what they are, it is not what they represent, conceptually.) Using this result, we refine the definition of proposals to be sets of updates-as-states:

Definition 13 (Proposal) A proposal A based on the model $\langle \mathbf{W}, I \rangle$ is a set of updates-as-states based on $\langle \mathbf{W}, I \rangle$, i.e., $A \subseteq \wp\mathbf{W}$. Let Π denote the set of all proposals (based on a given model).

A proposal is the same kind of object as a proposition in InqB , i.e., a set of states, but, crucially, it represents a different kind of object, namely, a set of update functions. Furthermore, such objects are interpreted as proposals:

Definition 14 (The Proposal View) A speaker, in uttering a sentence denoting a proposal A , proposes to execute any one of the update functions in A .

Frankly, not much is gained by spelling out the Proposal View: we take proposing as a primitive semantic notion. If we assume the Proposal View, then updates contained in another update, i.e., non-maximal updates-as-states, do make a difference. Hence, unlike propositions in InqB , proposals need not be downward (or upward) closed; they are defined as *unrestricted* sets of updates.

Besides the Proposal View, we need to settle on either a dynamic or a static perspective of the semantics, before we can pin down the semantic operation underlying conjunction. We aim for a dynamic semantics, i.e., one in which a conjunction is interpreted as a sequence of proposals. Like the Proposal View, this dynamic perspective is a choice. (It is motivated by the attractive outlook of connecting inquisitive semantics with dynamic semantics and dynamic epistemic logic.) The semantics we define will derive from these two assumptions.

3.2 Algebraic characterisation

Unlike in the case of InqB , the algebraic structure we define on the set of meanings cannot be a lattice, because the absorption laws must not hold. Instead, there will be two independent orders on proposals such that, at best, the addition operation gives the join with respect to one order, and multiplication gives the meet with respect to the other. Because of this, it does not matter whether we start by defining the operations, or by defining the orders, so long as we motivate the definitions according to the Proposal View. We start by defining the operations, for didactic reasons.

The multiplication operation (\otimes) should underly a dynamic conjunction of sentences, i.e., it should be a sequence operation on proposals. Meanings, spelled out according to the Proposal View, behave as follows under dynamic conjunction:

Observation 2 (Behaviour of conjunction) *If one proposes to execute any of the updates in A and (subsequently) proposes to execute any of the updates in B , one proposes to execute any two updates, of which the first in A and the second in B .*

If we think of updates as functions, then performing two updates f and g amounts to performing their composition $f \bullet g = \{\langle s, s'' \rangle : \exists s'. \langle s, s' \rangle \in f \text{ and } \langle s', s'' \rangle \in g\}$. If the functions are distributive and eliminative, as we have assumed, then their composition $f \otimes g$ is also distributive and eliminative. Hence, it too can be represented by a static object, i.e., a state. If a is the state corresponding to f , and b to g , then the state for $f \otimes g$ is given by $a \cap b$. Therefore, on *sets* of states, i.e., proposals, we define multiplication as *pointwise* intersection:

Definition 15 (Multiplication)

$$A \otimes B = \{a \cap b : a \in A, b \in B\}.$$

The proposal $\{\mathbf{W}\}$ is the identity element for multiplication, i.e., for all $A \in \mathbf{\Pi}$, $A \otimes \{\mathbf{W}\} = A$. Multiplication (as pointwise intersection) is associative and commutative. It is not idempotent, which is a consequence of adopting the Proposal View combined with our decision to treat conjunction as sequence: if a proposal, containing multiple updates, is made twice, a different update can be chosen the first and the second time, and both of them executed, giving a different result than if the proposal had been made only once. These properties imply that the set of proposals with the multiplication operation and its identity element form a *commutative monoid*:

Fact 4 $\langle \mathbf{\Pi}, \otimes, \{\mathbf{W}\} \rangle$ *is a commutative monoid, i.e.:*

1. $A \otimes \{\mathbf{W}\} = A$
2. $A \otimes (B \otimes C) = (A \otimes B) \otimes C$
3. $A \otimes B = B \otimes A$

Because multiplication is not idempotent, it cannot give the meet with respect to any partial order (the non-idempotency would be in conflict with the reflexivity of the order). However, commutative monoids come with a partial order, called the *divisibility order*, with respect to which multiplication *would have* given the meet, had it been idempotent.

Definition 16 (Divisibility order) $A \leq_{\otimes} B$ iff $\exists C. B \otimes C = A$.

This can be read as follows: $A \leq_{\otimes} B$ iff A can be \otimes -decomposed, i.e., factorized, into B and some other proposal C , i.e., iff B is a divisor of A .

The addition operation (\oplus) should correspond to disjunction. We spell out the Proposal View to see how proposals behave under disjunction:

Observation 3 (Behaviour of disjunction) *If one proposes to execute any of the updates in A or any of the updates in B , one proposes to execute any of the updates in $A \cup B$.*

Hence, we define our addition operator as ordinary set union:

Definition 17 (Addition) $A \oplus B = A \cup B$.

The proposal \emptyset is the identity element for addition, and addition (as union) is associative, commutative and idempotent, so we have:

Fact 5 $\langle \mathbf{II}, \oplus, \emptyset \rangle$ is a commutative, idempotent monoid, i.e.:

1. $A \oplus \emptyset = A$
2. $A \oplus (B \oplus C) = (A \oplus B) \oplus C$
3. $A \oplus B = B \oplus A$
4. $A \oplus A = A$

Every commutative, idempotent monoid has a partial order with respect to which it is a join-semilattice, and the addition operation a join operator. The order, which we call the *additive order*, is defined analogously to the divisibility order, but with \oplus instead of \otimes .

Definition 18 (Additive order) $A \geq_{\oplus} B$ iff $\exists C. B \oplus C = A$ (iff $A \oplus B = A$ iff $B \subseteq A$).

We can say that $A \geq_{\oplus} B$ iff A can be \oplus -decomposed into B and some other proposal C . Because \oplus is idempotent, this proposal C can be A itself, allowing for the more common definition in terms of $A \oplus B = A$, parenthesized above. In our case, this can be further simplified: the additive order is the inverse of set inclusion.

Fact 6 $\langle \mathbf{II}, \geq_{\oplus} \rangle$ is a join-semilattice, with \oplus as join.

As an encouraging sign that the Proposal View is on the right track, the present definitions of multiplication and addition ensure that the absorption laws do not hold, e.g., it is not generally the case that $A \oplus (A \otimes B)$ equals A . However, addition and multiplication do interact in the following ways. First, \emptyset , the identity element for \oplus , is an annihilator for \otimes , i.e., $\emptyset \otimes A = A \otimes \emptyset = \emptyset$. Second, \otimes (as pointwise intersection) distributes over \oplus (as union). These properties imply that the two monoids together form a *commutative, idempotent semiring*, i.e., a semiring with the additional properties that multiplication is commutative and addition is idempotent.

Fact 7 (Algebraic characterisation of Π)

$\langle \Pi, \oplus, \otimes, \emptyset, \{\mathbf{W}\} \rangle$ is an idempotent semiring, i.e.:

1. $\langle \Pi, \oplus, \emptyset \rangle$ is a commutative, idempotent monoid;
2. $\langle \Pi, \otimes, \{\mathbf{W}\} \rangle$ is a commutative monoid;
3. $A \otimes (B \oplus C) = (A \otimes B) \oplus (A \otimes C)$;
4. $\emptyset \otimes A = A \otimes \emptyset = \emptyset$.

3.3 Two orders: entailment and compliance

There are two orders on the set of proposals, the additive order (\geq_{\oplus}) and the divisibility order (\leq_{\otimes}). If we associate entailment with the additive order, then entailment will allow for \oplus -introduction, but not for \otimes -elimination. If we associate entailment with the divisibility order, entailment will allow for \otimes -elimination, but not for \oplus -introduction. Once again, the choice is guided by spelling out the Proposal View:

Observation 4 (Behaviour of entailment)

1. If one proposes to execute any update in $A \otimes B$, it follows that she proposes to execute any update in A .
2. If one proposes to execute any update in A , it does not follow that she proposes to execute any update in $A \oplus B$.

It follows that entailment on proposals should allow for \otimes -elimination, but not for \oplus -introduction. Hence, we associate entailment with the divisibility order, i.e., the order with respect to which \otimes is almost-but-not-quite a meet operation:

Definition 19 (Entailment)

For any $A, B \in \Pi$, A entails B , $A \vDash B$, iff $A \leq_{\otimes} B$ (iff $\exists C. B \otimes C = A$).

Note that, because multiplication is not idempotent, $A \vDash B$ does not mean that after expressing A , expressing B is redundant: $A (= B \otimes C)$ need not be identical with $A \otimes B (= (B \otimes C) \otimes B)$.

The additive order can be interpreted as follows. If $A \geq_{\oplus} B$, i.e., $B \subseteq A$, then all updates proposed by B are already proposed by A . If this is the case, we say that B *complies* with A , or that A *makes B compliant*. For clarity, we associate a new symbol with the additive order thusly interpreted:

Definition 20 (Compliance) A makes B compliant, $A \propto B$, iff $A \geq_{\oplus} B$ (iff $B \subseteq A$).

Compliance may be an important notion for describing discourse coherence. Consider the following range of responses to an initiative:

Observation 5

John, Mary or Bob will come.

1. Yes / #actually, John will come.
2. Yes / #actually, John or Mary will come.
3. Yes / #actually, John, Mary or Bob will come.
4. #Yes / actually, Charlie will come.

The unacceptability of ‘actually’ in the first three responses indicates that, unlike the last response, the first three maintain discourse coherence. Although we have not given the semantics yet, we can see by translating disjunction as addition that the first three are also exactly the ones that *comply* with the initiative, i.e.:

Fact 8 $\llbracket p \vee q \vee r \rrbracket \propto \llbracket p \rrbracket, \llbracket p \vee q \rrbracket, \llbracket p \vee q \vee r \rrbracket; \llbracket p \vee q \vee r \rrbracket \not\propto \llbracket s \rrbracket.$

This prompts us to formulate the following tentative generalization:

Conjecture 1 (Discourse coherence and compliance)

A sequence of proposals A_1, \dots, A_n is coherent iff $A_i \propto A_{i+1}$ for all $i, 0 \leq i \leq n - 1$.

But let us get back to defining the semantics.

3.4 Semantics

The relative pseudo-complement $A \oslash B$, if it exists, is the unique entailment-maximal (weakest) proposal C such that $A \otimes C \vDash B$. As can be seen from the following example, there is not always a unique such proposal (example due to F. Roelofsen).

Fact 9 *Let $A = \{\{00, 01\}, \{10, 11\}\}$ and $B = \{\{00\}, \{01\}, \{10\}, \{11\}\}$. Then there are three \vDash -maximal proposals C such that $A \otimes C \vDash B$, namely $C_1 = \{\{00, 10\}, \{01, 11\}\}$, $C_2 = \{\{00, 11\}, \{01, 10\}\}$ and $C_3 = C_2 \oplus C_1$, none of which entail another.*

So far, all definitions followed from a particular stance on meaning, combined with general algebraic concerns. Hence, the lack of a unique entailment-maximal proposal C such that $A \otimes C \vDash B$ is not a technical shortcoming, but a conceptual challenge - how are we to make sense of this? Our suggestion (but there may be multiple that make sense) is to interpret the existence of multiple entailment-maximal proposals as a source of indeterminacy or choice, just like ordinary disjunction of proposals. To capture this idea, let $A \oslash B$ denote the *compliance-maximal* proposal among the various, non-unique entailment-maximal proposals. In the example, this proposal is C_3 . In general, it is given by the union (join) of all entailment-maximal proposals C such that $A \otimes C \vDash B$. We will define this *compliance-maximal relative pseudo-complement* as follows:

Definition 21 (Compliance-maximal relative pseudo-complement)

$$A \oslash B = \bigcup \text{MAX}_{\vDash} \{C : A \otimes C \vDash B\}.$$

Currently, we have not yet discovered a more constructive definition, i.e., one that tells us how to construct the proposal $A \oslash B$. All we know at present is that a unique such proposal exists for arbitrary A and B . And as long as there exist only countably many proposals, as in the propositional case, it can be found.

In sum, we have the following operations:

Definition 22 (Basic operations on Π)

1. $A \oplus B = A \cup B$;
2. $A \otimes B = \{a \cap b : a \in A, b \in B\}$;
3. $A \odot B = \bigcup \text{MAX}_{\models} \{C : A \otimes C \models B\}$.

As before, we associate the basic operations with the logical connectives to obtain a semantics for the syntax of propositional logic, and this gives us *unrestricted inquisitive semantics* (**InqU**).

Definition 23 (Unrestricted inquisitive semantics (InqU**))**

For π a proposition letter, φ and ψ formulae:

1. $\llbracket \pi \rrbracket = \{\{w : w(\pi) = 1\}\}$;
2. $\llbracket 0 \rrbracket = \emptyset$;
3. $\llbracket \perp \rrbracket = \{\emptyset\}$;
4. $\llbracket \varphi \vee \psi \rrbracket = \llbracket \varphi \rrbracket \oplus \llbracket \psi \rrbracket$;
5. $\llbracket \varphi \wedge \psi \rrbracket = \llbracket \varphi \rrbracket \otimes \llbracket \psi \rrbracket$;
6. $\llbracket \varphi \rightarrow \psi \rrbracket = \llbracket \varphi \rrbracket \odot \llbracket \psi \rrbracket$.

Note that there are two contradiction-like proposals. $\llbracket 0 \rrbracket = \emptyset$ is the proposal to execute any one of zero updates, which is impossible, while $\llbracket \perp \rrbracket = \{\emptyset\}$ proposes to perform an update with inconsistent information. We need both 0 and \perp lest we lose functional completeness, because no composition of formulas $\varphi \neq 0$ denotes \emptyset , and no composition of formulas $\varphi \neq \perp$ denotes $\{\emptyset\}$. The former proposal we need as an identity element for join (which gives us a semiring structure), the latter is necessary to define negation.

Our definition of the semantics is equivalent to the definition in (Ciardelli et al., 2009), apart from the definition of implication (and apart from some technical differences in how empty sets are treated). Their definition of implication $\llbracket \varphi \rightarrow \psi \rrbracket$ gives the union over *all* proposals C such that $\llbracket \varphi \rrbracket \otimes C \models \llbracket \psi \rrbracket$, i.e., not just the entailment-maximal ones.

3.5 The Challenge

In **InqU**, without the absorption laws, $\llbracket p \vee q \rrbracket$ and $\llbracket p \vee q \vee (p \wedge q) \rrbracket$ are not equivalent. As the reader can easily verify, these formulas are assigned the proposals depicted in figure 3. This shows that **InqU** resolves the Challenge, i.e., **InqU** is a suitable semantic foundation for an account of the contrast in observation 1. Indeed, beyond laying the semantic foundation, **InqU** not merely *allows for* an explanation of observation 1; in fact it already predicts it, via conjecture 1 (that discourse coherence depends on compliance):

Fact 10 $\llbracket p \vee q \vee (p \wedge q) \rrbracket \propto \llbracket p \wedge q \rrbracket$; $\llbracket p \vee q \rrbracket \not\propto \llbracket p \wedge q \rrbracket$.

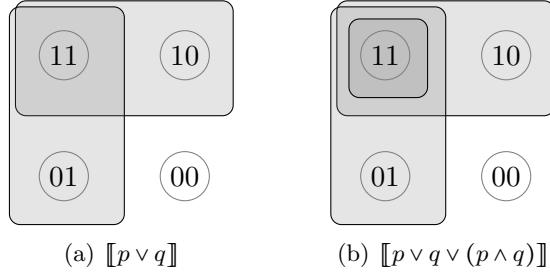


Figure 3: The proposals of the Challenge, according to InqU.

3.6 Relation to InqB: From proposals to propositions

Dialogue is not just about proposing updates; it is also (perhaps ultimately) about *information exchange*. Assuming that the speaker is sincere in uttering a proposal A , her proposal to execute any of the updates in A , provides the information that all updates in A are compatible with her world knowledge. Hence, if the speaker’s knowledge is true, the actual world is for sure in one of the states resulting from those updates. Furthermore, if a proposal A is intended to result in an action, i.e., an actual update, then meeting this intention requires that sufficient information is provided to establish for one of the updates that the resulting state will still contain the actual world.

Since we represent updates as states, we can define the *informative content* and *inquisitive content* of a proposal (provided the speaker is sincere and her knowledge is true) as follows:

Definition 24 (Informative and inquisitive content of proposals)

Expressing a proposal A provides the information that the actual world is in $\cup A$ (its informative content), and it requests sufficient information to locate the actual world in one of the states in A (inquisitive content).

We can represent this informative and inquisitive content of a proposal A in a separate semantic object associated with the proposal, written $\pi(A)$, which we may call its *proposition*. Definition 24 can then be reformulated in terms of propositions, and it would read precisely as the Basic View (definition 5).

Just like it followed from the Basic View, it follows from definition 24 that the proposition $\pi(A)$ of a proposal A is a downward-closed set of states. Hence, we can obtain $\pi(A)$, representing the informative and inquisitive content of a proposal A , as follows:

Fact 11 (From proposal to proposition)

Let A be a proposal, and $\pi(A)$ the proposition representing its informative and inquisitive content. Then $\pi(A) = \{s : \exists s' \in A. s \subseteq s'\}$.

We could now tread in Roelofsen’s (2011) footsteps and define an algebraic semantics for the set of propositions corresponding to the set of proposals. The result will again be a Heyting algebra, and the resulting semantics equivalent with InqB. In sum, InqB, a semantics of information exchange, can be derived from InqU, a semantics of proposals.

4 Conclusion and outlook

We have compared two implementations of inquisitive semantics with respect to their adequacy as a semantic foundation for a theory of discourse coherence. *Basic inquisitive semantics*, InqB , models providing and requesting information (Roelofsen, 2011); *unrestricted inquisitive semantics*, InqU , models proposals to update the common ground (Ciardelli et al., 2009). We have illustrated with a simple example that InqU , but not InqB , can be a semantic foundation for an account of discourse coherence. Motivated by this, we have redefined InqU from scratch, to increase our conceptual and technical understanding of the formalism. We have assumed a view of meanings as proposals and aimed for a dynamic notion of meaning. Based on this conception of meaning, the semantics could be defined driven by general algebraic concerns. The algebraic backbone of InqU turned out to be a commutative, idempotent semiring, and this gave rise to an entailment order and a compliance order. The latter order we have tentatively linked with discourse coherence. Finally, we have shown how InqB can be derived from InqU , both technically and conceptually.

Our definition of InqU differs from that in (Ciardelli et al., 2009) only substantially with respect to the clause for implication. Lacking a relative pseudo-complement operation, we combined the two orders on meanings to define implication as a compliance-maximal relative pseudo-complement. Although this decision followed from a particular conceptual choice, we still regard it as an improvement over the clause in (Ciardelli et al., 2009), which was defined in a much more ad-hoc fashion. Despite this difference, it seems that a large portion of the empirical coverage of InqU , which concerns in particular the modal ‘might’ and its interaction with disjunction and conjunction, can be maintained. Nevertheless, a more detailed comparison is necessary.

Conjecture 1 is intended as but an illustration of the link between discourse coherence and the compliance order on proposals. There may be many instances of coherent discourse that the notion of compliance, as we have defined it, does not capture. For instance, it seems that responding to a question with an easier-to-resolve subquestion should be allowed (Groenendijk & Roelofsen, 2009). Since compliance corresponds to a natural order on the algebraic structure, we think that a more general account of discourse coherence is best built on top of the current notion of compliance, rather than making the notion itself more complex.

Finally, we hope that the algebraic characterisation of InqU will help to link inquisitive semantics to other formalisms. In particular, idempotent semirings (or dioids) form the core of a Kleene algebra, which underlies the set of actions/transitions in propositional dynamic logic (PDL; see Eijk & Stokhof, 2006 for a recent overview). Exploring this link could lead to a transfer of many interesting results, proofs, and concepts. In particular, the connection with PDL could facilitate the integration of inquisitive semantics with dynamic epistemic logic and non-eliminative update functions required for belief revision.

References

- Benthem, J. van. (1989). Semantic parallels in natural language and computation. In H. D. Ebbinghaus, J. Fernandez-Prida, M. Garrido, & D. Lascar (Eds.), *Logic colloquium, granada, 1987* (p. 31375). Elsevier, Amsterdam.
- Ciardelli, I. (2010). A first-order inquisitive semantics. In M. Aloni, H. Bastiaanse, T. de Jager, & K. Schulz (Eds.), *Logic, language, and meaning: Selected papers from the seventeenth Amsterdam Colloquium*. Springer.
- Ciardelli, I., Groenendijk, J., & Roelofsen, F. (2009). Attention! *Might* in inquisitive semantics. In S. Ito & E. Cormany (Eds.), *Proceedings of semantics and linguistic theory (SALT XIX)*.
- Ciardelli, I., & Roelofsen, F. (2011). Inquisitive logic. *Journal of Philosophical Logic*, 40(1), 55–94.
- Eijk, J. van, & Stokhof, M. (2006). The gamut of dynamic logics. In D. Gabbay & J. Woods (Eds.), *The handbook of the history of logic. volume 6: Logic and modalities in the twentieth century* (p. 499-600). Elsevier.
- Groenendijk, J., & Roelofsen, F. (2009). Inquisitive semantics and pragmatics. In J. M. Larrazabal & L. Zubeldia (Eds.), *Meaning, content, and argument: Proceedings of the ILCLI international workshop on semantics, pragmatics, and rhetoric*.
- Roelofsen, F. (2011). Algebraic foundations for inquisitive semantics. In H. van Ditmarsch, J. Lang, & J. Shier (Eds.), *Proceedings of the third international conference on logic, rationality, and interaction* (pp. 233–243). Springer-Verlag.